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Contra p*gα- cleavability

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Abstract.

K.Geetha & M.Vigneshwaran (2016) [5] introduced and studied the concept of contra p*gα continuous function and strongly p*gα closed function.
In this paper ,we used these functions to study the concept of cleavability over these special topological spaces (P ultra Hausdorff , P-normal , Strongly compact , P-Ts , strongly p*gα-closed) spaces.as following :

1- If $\mathcal{P}$ is a class of topological spaces with certain properties and if X is cleavable over $\mathcal{P}$,then X $\notin \mathcal{P}$.
2- If $\mathcal{P}$ is a class of topological spaces with certain properties and if Y is cleavable over $\mathcal{P}$ then Y $\notin \mathcal{P}$.
3- If $\mathcal{P}$ is a class of topological spaces with certain properties and if Y is cleavable over $\mathcal{P}$ then X $\in \mathcal{P}$.

1-Introduction

Arhngel' Skii. A (1985) [1]introduced different types of cleavability(originally named splitability) as following : A topological space X is said to be cleavable over a class of spaces $\mathcal{P}$ if for $A \subset X$ there exists a continuous mapping f: X → Y ∈ $\mathcal{P}$ such that $f^{-1}f(A) = A$, f(X)=Y. . Throughout this paper (X, τ), (Y, σ) will always denote the topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let A be a Subset of (x, τ), cl (A) and
Int (A) denote the closure and interior of A, subset of (x, τ), cl (A) and Int (A) denote the closure and interior of A, A subset A of a space (X, τ) is called regular open (resp. regular closed) if A = int(cl(A)) (resp. A = cl(int(A))). respectively.

Keywords: contra p*gα −cleavability, p*gα− cleavability, contra-p*gα- point wise cleavability , contra-p*gα - absolutely cleavable, p*gα -irresolute cleavability.

2. Preliminaries:

Now we recall some definitions which we needed in this paper

Definition 2.1.[4]

Let(X, τ) be a topological space. A subset A of the space X is said to be Preopen if A ⊆ int(cl(A)) and preclosed if cl(int(A)) ⊆ A.

The intersection of all preclosed sets containing A is called preclosure of A and is denoted by pcl(A).

Definition 2.2

Let(X, τ) be a topological space. A subset A ⊆ X is said to be

p*gα-closed .[5] if pcl(A) ⊆ (U) whenever A ⊆ U and U is*gα-open in X. The complements is called open sets.

Definition 2.3.

A topological space (X,τ) is called A P-Ts space [3] if every p*gα -closed set is closed.

Theorem 2.1. [5]

Let (X, τ) be a topological space :

(1) A subset A of (X ,τ) is regular open ⟺A is opened p*gα closed.

(2) A subset A of (X, τ) is open and regular closed then A is p*gα closed.

Theorem 2.2. [3]

Every closed set in a topological pace (X ,τ) is p*gα-closed.

Definition 2.4

A function f: (X, τ)⟶(Y,σ) is called contra- p*gα-continuous if f^{-1}(V) is p*gα-open (respectively p*gα closed) in (X, τ) for every closed (respectively open) set V in (Y, σ).

Example 2.1.Let X = {a, b, c} = Y, t = { φ, {a}, X} and σ = { φ, {b, c}, Y}. Then the identity function

f: (X, τ)⟶(Y,σ) is contra-p*gα continuous function, since for the closed (respectively open) set V = {b} in (Y,σ), f^{-1}(V) = {b} is p*g- open (respectively p*gα closed) in (X, τ).

Example2.2

Let X = {1, 2, 3} = Y, τ = {φ, {1}, X} and σ = {φ, {1}, {2, 3}, Y}. Define

f : (X, τ) → (Y, σ) by f (1) = 2; f (2) = 3 and f (3) = 1 .Then f is contra p*gα continuous function, since for the closed (respectively open) set V = {2} in (Y,σ), f^{-1}(V) = {2} is p*g- open (respectively p*g closed) in (X, τ).

Definition2.5[5]

A function f: X⟶Y from a topological space X into a topological space Y is said to be p*gα -irresolute if f^{-1} (V) is p*gα -closed in X for every p*gα - closed set V in Y.

3- contra- p*gα -cleavability

Definition 3.1

A topological space X is said to be a contra- p*gα (resp . contra-p*gα irresolute) cleavable over a class of spaces $\mathcal{P}$,if for any subset A of X, there exists a contra - p*gα - (resp . contra-p*gα irresolute) –continuous mapping f : X→Y , such that $f^{-1}f$(A) = A. and $f(X) = Y$.

Definition 3.2

A topological spaces is said to be a contra-p*gα -pointwise cleavable over a class of spaces $\mathcal{P}$. if for every point $x \in X$ there exists an injective contra-p*gα - continuous mapping f : X→Y , such that $f^{-1}f$ { x}={ x }.

Remark 3.1

By a contra-p*g$\boldsymbol{\alpha}$ - pointwise cleavable ,we mean that a contra -p*g$\boldsymbol{\alpha}$ - continuous function $\boldsymbol{f}$: $\boldsymbol{X}$→$\boldsymbol{Y} \in \boldsymbol{\mathcal{P}}$ is an injective and a contra p*g$\boldsymbol{\alpha}$ continuous.

Definition3.3

A topological space X is said to be a contra -p*gα - absolutely cleavable over a class of spaces $\mathcal{P}$,If for any subset A of X , there exists an injective contra– p*gα continuous mapping f : X→Y , such that $f^{-1}f$(A) = A .

Remark 3.2

If f is an open (.closed) a contra- – p*gα - continuous mapping , we mean that

is an open (.closed) contra– p*gα absolutely cleavable over $\mathcal{P}$ .X

Definition 3.4[5]

A topological space (X, τ) is said to be P-Hausdorff if for each pair of distinct points x and y in X there exist disjoint p*gα open sets A and B of x and y respectively.

Definition3.5 [6]

A topological space X is called:

(a) ultra Hausdroff space if every pair of distinct points of x and y in X there

exist disjoint clopen sets U and V in X containing x and y respectively.

(b) ultra-normal if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.

Proposition 3.1

Let X be a contra-p*gα- pointwise cleavable over class of ultra Hausdroff space $\mathcal{P}$, then X is P- Hausdorff space .

Proof:

Suppose $x \in X$, then there exists an ultra Hausdorff space Y and a contra-p*gα-

continuous mapping $f: X \rightarrow Y \in \mathcal{P}$ such that $f^{-1}f(x) = \{x\}$. This implies that

for every y $\in Y$ with $x \neq y$,we have $f(x) \neq f(y)$. Since Y is an ultraHausdorff , so there exists two clopen sets G and H such that $f(x) \in G$,

$\boldsymbol{f}(\boldsymbol{y}) \in \boldsymbol{H}$ and $\boldsymbol{G} \bigcap \boldsymbol{H} = \emptyset$, then $\boldsymbol{f}^{-1}\boldsymbol{f}(\boldsymbol{x}) \in \boldsymbol{f}^{-1}(\boldsymbol{G})$, $\boldsymbol{f}^{-1}\boldsymbol{f}(\boldsymbol{y}) \in \boldsymbol{f}^{-1}(\boldsymbol{H})$, this implies

that $\boldsymbol{x} \in \boldsymbol{f}^{-1}$(G) , $\boldsymbol{y} \in \boldsymbol{f}^{-1}$(H) , since $\boldsymbol{f}$ is a contra-p*g$\boldsymbol{\alpha}$- continuous , so $\boldsymbol{f}^{-1}(\boldsymbol{G})$, $\boldsymbol{f}^{-1}(\boldsymbol{H})$ are p*g$\boldsymbol{\alpha}$ open sets of $\boldsymbol{X}$ and

$\boldsymbol{f}^{-1}(\boldsymbol{U}) \bigcap \boldsymbol{f}^{-1}(\boldsymbol{V}) = \boldsymbol{f}^{-1}(\boldsymbol{U} \bigcap \boldsymbol{V}) = \boldsymbol{f}^{-1}(\emptyset) = \emptyset$. then $\boldsymbol{X}$ is P-Hausdorff space

Definition 3.6[5]

A topological space (X, τ) is called P-normal if each pair of nonempty disjoint closed sets can be separated by disjoint p*gα-open sets.

Proposition 3.2

Let $\boldsymbol{X}$ be a closed contra-p*g$\boldsymbol{\alpha}$ absolutely cleavable over a class of ultra normal spaces $\boldsymbol{\mathcal{P}}$, then $\boldsymbol{X}$ is p- normal.

Proof:

Suppose F_1, F_2 be two non-empty disjoint closed sets of X , then there exists an injective closed a contra-p*gα- continuous mapping $f:X \longrightarrow Y \in \mathcal{P}$ such that $\boldsymbol{f}^{-1}\,\boldsymbol{f}(\boldsymbol{F}_1)=\boldsymbol{F}_1$, $\boldsymbol{f}^{-1}\boldsymbol{f}(\boldsymbol{F}_2)=\boldsymbol{F}_2$. Since $\boldsymbol{f}$ is closed and injective , then $\boldsymbol{f}\,(\boldsymbol{F}_1)$, $f\,(F_2)$ are two non-empty disjoint closed sets of Y, since Y is an ultra normal space , so there exist two clopen sets $\boldsymbol{E_1}$, $\boldsymbol{E_2}$ such that $\boldsymbol{f}\,(\boldsymbol{F}_1) \subset \boldsymbol{E_1}$, $\boldsymbol{f}\,(\boldsymbol{F}_2) \subset \boldsymbol{E_2}$ and $\boldsymbol{E_1} \bigcap \boldsymbol{E_2} = \emptyset$, then $\boldsymbol{f}^{-1}\boldsymbol{f}(\boldsymbol{F}_1) \subset \boldsymbol{f}^{-1}(\boldsymbol{E_1})\ \boldsymbol{f}^{-1}\boldsymbol{f}(\boldsymbol{F}_2) \subset \boldsymbol{f}^{-1}(\boldsymbol{E_2})$, this implies that $\boldsymbol{f}^{-1}(\boldsymbol{E_1})$, $\boldsymbol{f}^{-1}(\boldsymbol{E_2})$ are contra-p*g$\boldsymbol{\alpha}$ open sets of $\boldsymbol{X}$ and $f^{-1}(E_1) \bigcap f^{-1}(E_2) = f^{-1}(E_1 \bigcap E_2) = f^{-1}(\emptyset) = \emptyset$ .Hence X is P- normal space .

Definition 3.7

A topological space (X, τ) is said to be p*gα-connected if X cannot be written as the disjoint union of two non-empty p*gα-open sets.

Proposition 3.3

Let **X** be p*g$\boldsymbol{\alpha}$ connected spaces which is a contra- cleavable over a class of $\boldsymbol{\mathcal{P}}$, then **Y** is conncted .

Proof: Suppose $\boldsymbol{Y}$ is not connected space , then $\boldsymbol{Y} = \boldsymbol{U} \bigcup \boldsymbol{V}$, where $\boldsymbol{U}$, $\boldsymbol{V}$ are disjoint non empty clopen sets of $\boldsymbol{Y}$, then there exists an injective contra- continuous mapping $\boldsymbol{f}:\boldsymbol{X} \longrightarrow \boldsymbol{Y} \in \boldsymbol{\mathcal{P}}$ such that $\boldsymbol{f}^{-1}\boldsymbol{f}\{\boldsymbol{f}^{-1}(\boldsymbol{U})\} = \boldsymbol{f}^{-1}(\boldsymbol{U})$, $f^{-1}f\{f^{-1}(V)\} = f^{-1}(V)$, since $Y = U \bigcup V$, then $f^{-1}(Y) = f^{-1}(U \bigcup V)$ this implies that $X = f^{-1}(U) \bigcup f^{-1}(V)$, since f is a contra - p*gα –continuous function ,then $f^{-1}(U)$, $f^{-1}(V)$ are non empty disjoint p*gα -open sets in X , which contradicts that X is p*gα connected , thee for Y is conncted .

Definition 3.8[5]

A topological space (X ,τ) is called Strongly compact if every preopen cover of X has a finite sub cover.

Theorem 3.2. [5]

Every strongly p*gα -closed space (X, t) is a compact S-closed space.

Proposition3.4

Let X be a strongly compact is a contra-p*gα a closed cleavable space over a class of spaces $\mathcal{P}$, then Y is compact space.

Proof :

Suppose $\{V_i\}_{i\in I}$ be preopen open cover of $\boldsymbol{Y}$, and there exists a contra-p*g$\boldsymbol{\alpha}$ continuous mapping $\boldsymbol{f}:\boldsymbol{X}\longrightarrow\boldsymbol{Y}\in\boldsymbol{\mathcal{P}}$, such that $\boldsymbol{f}^{-1}\boldsymbol{f}\{\boldsymbol{f}^{-1}\{V_i\}_{i\in I}\}=\boldsymbol{f}^{-1}\{V_i\}_{i\in I}$, since $\boldsymbol{f}$ is contra-p*g$\boldsymbol{\alpha}$ -continuous , then $\boldsymbol{f}^{-1}\{V_i\}_{i\in I}$ is a p*g$\boldsymbol{\alpha}$ -closed cover of $\boldsymbol{X}$. but $\boldsymbol{X}$ is strongly p*gα -closed , so there exists a finite sub cover $\{\boldsymbol{f}^{-1}(V_1),\ldots,\boldsymbol{f}^{-1}(V_n)\}$ of $\boldsymbol{X}$, such that

$\mathbf{X}\subset\bigcup_{i=1}^{n}\{\boldsymbol{f}^{-1}(V_i)\}$,since $\boldsymbol{f}\boldsymbol{f}^{-1}(V_i)=V_i$, So $\{V_1,\ldots,V_n\}$ is a finite open Sub cover of Y. Therefore Y is a compact.

Definition 3.8

A topological space (X ,τ) is called p*gα-compact if every p*gα-open cover of X has a finite subcover.

Proposition3.5

Let X be p*gα –compact is a contra-p*gα a cleavable over a class of spaces $\mathcal{P}$, then Y is strongly S-closed.

Proof:

Suppose $\{V_i\}_{i\in I}$ be an closed cover of Y , since X is a contra-p*gα cleavable, so there exists a contra p*gα continuous mapping $f:X\longrightarrow Y\in\mathcal{P}$, such that $f^{-1}f\{f^{-1}\{V_i\}_{i\in I}\}=f^{-1}\{V_i\}_{i\in I}$, since f is contra- p*gα -continuous , then

$f^{-1}\{V_i\}_{i\in I}$ is a p*gα -open cover of X . but X is p*gα - compact , so there exists a finite sub cover$\{f^{-1}(V_1), \ldots, f^{-1}(V_n)\}$ of X , such that $X \subset \bigcup_{i=1}^{n} \{f^{-1}(V_i)\}$, since $ff^{-1}(V_i) = V_i$, So $\{V_1, \ldots, V_n\}$ is a finite sub sub cover of $\boldsymbol{Y}$. Therefore $\boldsymbol{Y}$ is strongly S-closed.

Definition 3.9[2]

A topological space (X,τ) is said to be P-Ts space if every p*gα -closed set is closed.

Proposition 3.6

Let X be a p*gα -compact space is a contra-p*gα - cleavable space over a class of $\mathcal{P}$, then Y is strongly p*gα -closed.

Proof :

Suppose $\{V_i\}_{i\in I}$ be any p*gα -closed cover of Y , Since Y is P-Ts space, then exists a contra-p*g$\boldsymbol{\alpha}$ continuous mapping $\boldsymbol{f}:\boldsymbol{X} \longrightarrow \boldsymbol{Y} \in \boldsymbol{\mathcal{P}}$, such that $f^{-1}f\{f^{-1}\{V_i\}_{i\in I}\} = f^{-1}\{V_i\}_{i\in I}$, since f is contra-p*gα -continuous , then $f^{-1}\{V_i\}_{i\in I}$ is a p*gα –open cover of X . since X is p*gα -compact , so there exists a finite open sub cover $\{f^{-1}(V_1), \ldots, f^{-1}(V_n)\}$ of X , such that $X \subset \bigcup_{i=1}^{n} \{f^{-1}(V_i)\}$, since $ff^{-1}(V_i) = V_i$, So $\{V_1, \ldots, V_n\}$ is a finite sub cover of $\boldsymbol{Y}$. Then $\boldsymbol{Y}$ is strongly p*g$\boldsymbol{\alpha}$ -closed.

Definition 3.10[5]

A topological space (X,τ) is said to be strongly p*gα-closed if every p*gα-closed cover of X has a finite sub cover.

Proposition3.7

Let X be a strongly p*gα -closed is a p*gα irresolute- cleavable over a class of spaces $\mathcal{P}$, then Y is strongly p*gα -closed.

Proof :

Suppose $\{V_i\}_{i\in I}$ be any closed cover of Y , since X is p*gα irresolute cleavable, so there exists a p*gα irresolute continuous mapping $f:X \longrightarrow Y \in \mathcal{P}$, such that $f^{-1}f\{f^{-1}\{V_i\}_{i\in I}\} = f^{-1}\{V_i\}_{i\in I}$, since f is p*gα irresolute -continuous , then $\boldsymbol{f}^{-1}\{V_i\}_{i\in I}$ is a p*g$\boldsymbol{\alpha}$ -closed cover of $\boldsymbol{X}$ since $\boldsymbol{X}$ is strongly p*gα -closed, so there exists a finite sub cover

$\{\boldsymbol{f}^{-1}(V_1), \ldots, \boldsymbol{f}^{-1}(V_n)\}$ of $\boldsymbol{X}$, such that $\mathbf{X} \subset \bigcup_{i=1}^{n} \{\boldsymbol{f}^{-1}(V_i)\}$,

since $\boldsymbol{ff}^{-1}(V_i) = V_i$, So $\{V_1, \ldots, V_n\}$ is a finite sub cover of $\boldsymbol{Y}$. Then $\boldsymbol{Y}$ is strongly p*g$\boldsymbol{\alpha}$ -closed.

CONCLUSION

In this paper we have studied and proved these cases:

1) If $\mathcal{P}$ is a class of ultra Hausdorff spaces with certain properties and if X is a contra-p*gα - point wise cleavable over $\mathcal{P}$, then X is P- Hausdorff space, hence $X \notin \mathcal{P}$.

2) If $\mathcal{P}$ is a class of ultra normal spaces with certain properties and if X is a closed contra-p*gα absolutely cleavable over a class of spaces $\mathcal{P}$, then X is p- normal. Hence $X \notin \mathcal{P}$.

3) If $\mathcal{P}$ is a class of p*gα – connecte with certain properties and if X is a contra-p*gα cleavable over a class of spaces $\mathcal{P}$, Then X is conncted . Hence $X \notin \mathcal{P}$.

4) If $\mathcal{P}$ is a class of p*gα –compact with certain properties and if X is a contra-p*gα cleavable over a class of spaces $\mathcal{P}$, then Y is strongly p*gα –closed or strongly S-closed . Hence $Y \notin \mathcal{P}$

5) If $\mathcal{P}$ is a class of a strongly p*gα -closed with certain properties and if X is p*gα irresolute- cleavable over a class of spaces $\mathcal{P}$, Then X is strongly p*gα –closed. Hence $X \in \mathcal{P}$.

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