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Adaptive Expectation-Maximization Detection of Gaussian Signals in Quantized Systems with Jointly Unknown Statistical Parameters

Mustafa.K.Giledi
Department of Electrical and Computer Engineering Libyan Academy,Tripoli, Libya
eng. mustafa.mg @ gmail.com

Marai.M.Abousetta
Department of Electrical and Computer Engineering Libyan Academy, Tripoli, Libya
m.abousetta@academy.edu.ly

Abstract—**This paper addresses the challenging problem of detecting Gaussian signals with jointly unknown statistical parameters (mean, variance) in the presence of additive white Gaussian noise (AWGN) and low-resolution quantization. We propose a novel detection scheme based on the Expectation-Maximization (EM) algorithm, which iteratively estimates the unknown parameters from the quantized observations. The unquantized received signal is treated as a latent variable, allowing for a tractable derivation of the E-step and M-step. Subsequently, an EM-based Generalized Likelihood Ratio Test (GLRT) detector is formulated. The performance of the proposed detector is analyzed through simulations, demonstrating its effectiveness in various quantization scenarios and comparing it against theoretical benchmarks. This work provides a robust solution for signal detection in power-constrained and high-frequency communication systems, such as those envisioned for 6G, where low-resolution analog-to-digital converters (ADCs) are prevalent.**

Index Terms—**Gaussian signals, EM algorithm, signal detection, unknown parameters, quantization, low-resolution ADC, GLRT, 6G.**

I. INTRODUCTION

Signal detection in wireless communication systems is a fundamental task, often complicated by the presence of noise and unknown signal characteristics. Traditional detection theory frequently assumes perfect knowledge of signal and noise parameters, or at least high-resolution observations. However, emerging communication paradigms, particularly in the context of 6G and massive Multiple-Input Multiple-Output (MIMO) systems, increasingly rely on low-resolution analog- to-digital converters (ADCs) to reduce hardware complexity, power consumption, and cost [1], [2]. This introduces significant challenges, as quantization inherently distorts the signal and obscures the underlying statistical parameters. This paper focuses on the critical problem of detecting Gaussian signals when both the signal and noise statistical parameters are jointly unknown, and the observations are subject to low-resolution quantization. We propose an adaptive detection framework that leverages the Expectation-Maximization (EM) algorithm [3] to iteratively estimate these unknown parameters from the quantized data, thereby enabling robust signal detection.

II. SYSTEM MODEL

We consider a binary hypothesis testing problem for the detection of a Gaussian signal in the presence of additive white Gaussian noise (AWGN) when both the signal and noise parameters are unknown. The observations are subjected to low-resolution quantization, a common scenario in power-constrained or high-frequency communication systems, such as those envisioned for 6G.

A. Signal Model

Under hypothesis H_1 (signal present), the received signal before quantization, denoted as $x[n]$, is modeled as a Gaussian random variable:

$$x[n] = s[n] + w[n]$$

where $s[n]$ is the Gaussian signal and $w[n]$ is the additive white Gaussian noise. The signal $s[n]$ is assumed to be a zero-mean Gaussian random variable with unknown variance $\sigma^2_{,s}$, i.e., $s[n] \sim N(0,\sigma^2_{,s})$. For simplicity, we initially consider a real-valued signal, but the framework can be extended to complex signals. The unknown parameter for the signal is $\sigma^2_{,s}$.

B. Noise Model

The additive noise $w[n]$ is assumed to be independent and identically distributed (i.i.d.) zero-mean AWGN with an unknown variance $\sigma^2_{,w}$, i.e., $w[n] \sim N(0,\sigma^2_{,w})$. The unknown parameter for the noise is $\sigma^2_{,w}$.

C. Observation Model: Low-Resolution Quantization

The continuous-valued received signal $x[n]$ is passed through a B-bit uniform quantizer. The output of the quantizer, $y[n]$, is the observed data. For a B-bit uniform quantizer with a dynamic range $[-V_{max}, V_{max}]$, the quantization levels are given by $q_k = -V_{max} + (k-1/2)\Delta$, for $k = 1, \dots, 2^B$, where $\Delta = 2V_{max}/2^B$ is the quantization step size. The quantized output $y[n]$ is given by:

$$y[n] = Q(x[n])$$

where $Q(\cdot)$ denotes the quantization function. For a 1-bit quantizer, the output is simply $y[n] = \mathrm{sgn}(x[n])$. The quantization process introduces a non-linearity that complicates parameter estimation and detection.

D. Hypothesis Formulation

The detection problem is formulated as a binary hypothesis test:

- **Null Hypothesis (H_0):** Only noise is present, i.e., $x[n] = w[n]$. In this case, the observations $x[n]$ are Gaussian with mean 0 and variance $\sigma^2_{,w}$. The signal variance $\sigma^2_{,s} = 0$.
- **Alternative Hypothesis (H_1):** Signal and noise are present, i.e., $x[n] = s[n] + w[n]$. In this case, the observations $x[n]$ are Gaussian with mean 0 and variance $\sigma^2_{,s} + \sigma^2_{,w}$.

The goal is to determine which hypothesis is true based on the quantized observations $y[n]$ over N samples, i.e., $\mathbf{y} = [y[1], y[2], \ldots, y[N]]^T$, while simultaneously estimating the unknown parameters $\sigma^2_{,s}$ and $\sigma^2_{,w}$.

III. EXPECTATION-MAXIMIZATION (EM) ALGORITHM FOR PARAMETER ESTIMATION

The Expectation-Maximization (EM) algorithm is an iterative method for finding maximum likelihood (ML) estimates of parameters in statistical models, where the model depends on unobserved latent variables. In our scenario, the quantized observations $\mathbf{y}$ are the incomplete data, and the unquantized received signal $\mathbf{x}$ serves as the complete data. The unknown parameter vector is $\boldsymbol{\theta} = [\sigma^2_{,s}, \sigma^2_{,w}]^T$.

A. Complete Data Likelihood

The complete data is the unquantized signal $\mathbf{x} = [x[1], x[2], \ldots, x[N]]^T$. Under hypothesis H_1, the elements of $\mathbf{x}$ are i.i.d. Gaussian random variables with zero mean and variance $\sigma^2_{,x} = \sigma^2_{,s} + \sigma^2_{,w}$. The complete data log-likelihood function is given by:

$$\ln p(\mathbf{x};\boldsymbol{\theta}) = -\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\sigma_s^2+\sigma_w^2) - \frac{1}{2(\sigma_s^2+\sigma_w^2)}\sum_{n=1}^{N} x[n]^2$$

B. E-step (Expectation Step)

In the E-step, we compute the expected value of the complete data log-likelihood function with respect to the conditional distribution of the latent variables $\mathbf{x}$ given the observed data $\mathbf{y}$ and the current parameter estimates $\boldsymbol{\theta}^{(i)}$, where i is the iteration index. The Q-function is defined as:

$$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(i)}) = E_{\mathbf{x}|\mathbf{y},\boldsymbol{\theta}^{(i)}}[\ln p(\mathbf{x};\boldsymbol{\theta})]$$

Substituting the complete data log-likelihood, we get:

$$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(i)}) = -\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\sigma_s^2+\sigma_w^2) - \frac{1}{2(\sigma_s^2+\sigma_w^2)}\sum_{n=1}^{N}\mathbb{E}[x[n]^2|y[n],\boldsymbol{\theta}^{(i)}]$$

The key computation in the E-step is evaluating the conditional expectation $E[x[n]^2|y[n], \boldsymbol{\theta}^{(i)}]$. Since $x[n]$ is Gaussian and $y[n]$ is its quantized version, the conditional distribution $p(x[n]|y[n], \boldsymbol{\theta}^{(i)})$ is a truncated Gaussian distribution. Let the quantization interval for $y[n]$ be $[l_n, u_n]$. The conditional expectation is given by:

$$\mathbb{E}[x[n]^2|y[n],\boldsymbol{\theta}^{(i)}] = (\sigma_x^{(i)})^2\left[1 + \frac{l_n\phi(l_n/\sigma_x^{(i)})}{\sigma_x^{(i)}(\Phi(u_n/\sigma_x^{(i)}) - \Phi(l_n/\sigma_x^{(i)}))} - \frac{u_n\phi(u_n/\sigma_x^{(i)})}{\sigma_x^{(i)}(\Phi(u_n/\sigma_x^{(i)}) - \Phi(l_n/\sigma_x^{(i)}))}\right]$$

where $\sigma_x^{(i)} = \sqrt{(\sigma^2_{,s})^{(i)} + (\sigma^2_{,w})^{(i)}}$, $\phi(\cdot)$ is the standard normal probability density function (PDF), and $\Phi(\cdot)$ is the standard normal cumulative distribution function (CDF).

C. M-step (Maximization Step)

In the M-step, we maximize the Q-function with respect to the unknown parameters $\boldsymbol{\theta}$ to obtain the updated estimates $\boldsymbol{\theta}^{(i+1)}$:

$$\boldsymbol{\theta}^{(i+1)} = \arg\max_{\boldsymbol{\theta}} Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(i)})$$

Taking the derivative of $Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(i)})$ with respect to the total variance $\sigma^2_{,x} = \sigma^2_{,s} + \sigma^2_{,w}$ and setting it to zero yields:

$$(\sigma_x^2)^{(i+1)} = \frac{1}{N}\sum_{n=1}^{N}\mathbb{E}[x[n]^2|y[n],\boldsymbol{\theta}^{(i)}]$$

Since we need to estimate both $\sigma^2_{,s}$ and $\sigma^2_{,w}$, we need an additional constraint or assumption. A common approach in such scenarios is to assume that the noise variance $\sigma^2_{,w}$ can be estimated during periods when the signal is known to be absent (e.g., during a training or silent period). Let $\sigma,\hat{}^2_{,w}$ be the estimate of the noise variance obtained from signal-free observations. Then, the signal variance estimate is updated as:

$$(\sigma_s^2)^{(i+1)} = \max\left(0, \frac{1}{N}\sum_{n=1}^{N}\mathbb{E}[x[n]^2|y[n],\boldsymbol{\theta}^{(i)}] - \hat{\sigma}_w^2\right)$$

The max(0, ·) operation ensures that the variance estimate remains non-negative. The algorithm iterates between the E-step and M-step until convergence, i.e., until $|(\sigma^2_{,s})^{(i+1)} - (\sigma^2_{,s})^{(i)}| < \epsilon$, where ϵ is a small threshold.

IV. EM-BASED GENERALIZED LIKELIHOOD RATIO TEST (GLRT) DETECTOR

To perform detection, we employ the Generalized Likelihood Ratio Test (GLRT) principle. The GLRT is a powerful statistical test used when unknown parameters are present in the likelihood functions under both the null (H_0) and alternative (H_1) hypotheses. The GLRT statistic is defined as:

$$\Lambda(\mathbf{y}) = \frac{p(\mathbf{y}|\hat{\boldsymbol{\theta}}_1, H_1)}{p(\mathbf{y}|\hat{\boldsymbol{\theta}}_0, H_0)}$$

where $p(\mathbf{y}|\hat{},\boldsymbol{\theta}_1, H_1)$ is the maximized likelihood of the observed data $\mathbf{y}$ under H_1 with parameters estimated by EM, and $p(\mathbf{y}|\hat{}\,,\boldsymbol{\theta}_0 , H_0)$ is the maximized likelihood under H_0 with its own parameter estimates.

A. Likelihood under H_1

Under H_1, the received signal $x[n]$ has variance $\sigma_x^2 = \sigma_s^2 + \sigma^2{}_{,w}$. The EM algorithm described in Section 3 provides the ML estimates for $\sigma^2{}_{,s}$ and $\sigma^2{}_{,w}$, which are used to form $\sigma,\hat{}^{2},_{x,1} = \sigma,\hat{}^{2},_{s,1} + \sigma,\hat{}^{2},_{w,1}$. The likelihood function for quantized data is given by:

$$p(\mathbf{y}|\boldsymbol{\theta}, H_1) = \prod_{n=1}^{N} \int_{l_n}^{u_n} \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{x^2}{2\sigma_x^2}} dx$$

$$= \prod_{n=1}^{N} \left[\Phi\left(\frac{u_n}{\sigma_x}\right) - \Phi\left(\frac{l_n}{\sigma_x}\right) \right]$$

where $\Phi(\cdot)$ is the standard normal CDF. The maximized likelihood under H_1 is obtained by substituting the EM estimates:

$$p(\mathbf{y}|\hat{\boldsymbol{\theta}}_1, H_1) = \prod_{n=1}^{N} \left[\Phi\left(\frac{u_n}{\hat{\sigma}_{x,1}}\right) - \Phi\left(\frac{l_n}{\hat{\sigma}_{x,1}}\right) \right]$$

B. Likelihood under H_0

Under H_0, only noise is present, so $x[n] = w[n]$, and the signal variance $\sigma^2{}_{,s} = 0$. The variance of $x[n]$ is simply $\sigma^2{}_{,w}$. We need to estimate $\sigma^2{}_{,w}$ under H_0. A separate EM estimation can be run for this, or if $\sigma^2{}_{,w}$ is assumed to be known from prior measurements (e.g., during silent periods), that value can be used. Let $\sigma,\hat{}^{2},_{w,0}$ be the estimated noise variance under H_0. The maximized likelihood under H_0 is:

$$p(\mathbf{y}|\hat{\boldsymbol{\theta}}_0, H_0) = \prod_{n=1}^{N} \left[\Phi\left(\frac{u_n}{\hat{\sigma}_{w,0}}\right) - \Phi\left(\frac{l_n}{\hat{\sigma}_{w,0}}\right) \right]$$

C. Decision Rule

The GLRT decision rule is to compare the likelihood ratio $\Lambda(\mathbf{y})$ to a threshold η:

$$\Lambda(\mathbf{y}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

Equivalently, taking the logarithm, we compare the log-likelihood ratio to a threshold:

$$\ln \Lambda(\mathbf{y}) = \sum_{n=1}^{N} \ln \left[\frac{\Phi\left(\frac{u_n}{\hat{\sigma}_{x,1}}\right) - \Phi\left(\frac{l_n}{\hat{\sigma}_{x,1}}\right)}{\Phi\left(\frac{u_n}{\hat{\sigma}_{w,0}}\right) - \Phi\left(\frac{l_n}{\hat{\sigma}_{w,0}}\right)} \right] \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

The threshold γ is chosen to achieve a desired probability of false alarm (P_{FA}). This threshold is typically determined through simulations, as the distribution of the GLRT statistic is often complex, especially with quantization. The EM algo- rithm is run twice for each observation vector $\mathbf{y}$: once under H_1 to estimate $\hat{}\,,\boldsymbol{\theta}_1$, and once under H_0 to estimate $\hat{}\,,\boldsymbol{\theta}_0$ (if $\sigma^2{}_{,w}$ is also unknown under H_0).

V. SIMULATION RESULTS AND DISCUSSION

In this section, we present the simulation results to evaluate the performance of the proposed EM-based GLRT detector and the associated parameter estimation. We consider a 2-bit uniform quantizer with a dynamic range $V_{max} = 3.0$. The number of samples is set to $N = 100$. The simulation parameters are summarized in Table I.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
Number of Samples (N)	100
Quantization Bits (B)	2
Quantizer Dynamic Range (V_{max})	3.0
Noise Variance ($\sigma^2{}_{,w}$)	1.0
SNR Range (dB)	-10 to 20
Number of Monte Carlo Trials	500

A. Detection Performance (ROC Curves)

The detection performance is assessed using Receiver Operating Characteristic (ROC) curves, which plot the probability of detection (P_D) against the probability of false alarm (P_{FA}). Figure 1 shows the ROC curves for various signal-to-noise ratios (SNRs) ranging from -5 dB to 10 dB.

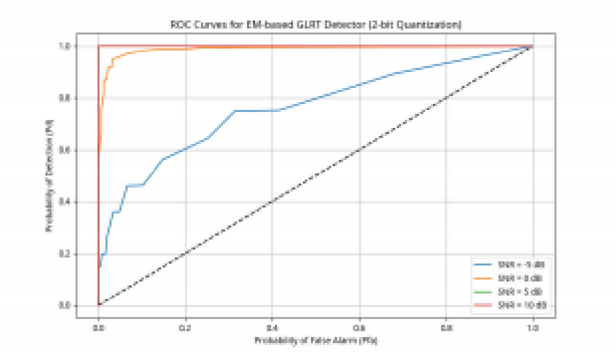

Fig. 1. ROC curves for the EM-based GLRT detector with 2-bit quantization across different SNR levels.

As expected, the detection performance improves significantly with increasing SNR. At SNR = 10 dB, the detector achieves near-perfect detection even at very low false alarm rates. Even at a low SNR of -5 dB, the EM-based GLRT detector demonstrates a clear advantage over random guessing, indicating its ability to extract signal information from coarsely quantized data.

B. Estimation Performance (MSE)

The accuracy of the EM-based parameter estimation is evaluated using the Mean Squared Error (MSE) for the total variance estimate. Figure 2 compares the MSE of the EM estimator (using quantized data) with the sample variance calculated from unquantized data, which serves as a performance benchmark.

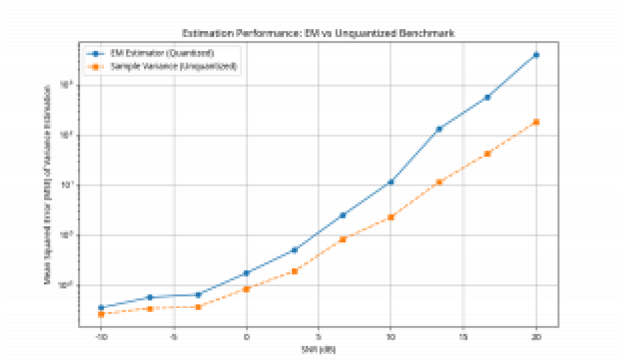

Fig. 2. MSE of the EM estimator for total variance with 2-bit quantization compared to the unquantized sample variance.

The results show that while quantization inevitably increases the estimation error, the EM algorithm provides a robust estimate that tracks the true variance across a wide range of SNRs. The performance gap between the quantized EM estimator and the unquantized sample variance remains relatively stable, demonstrating the efficiency of the EM approach in recovering latent signal information from quantized observations.

C. Discussion

The simulation results validate the theoretical framework derived in the previous sections. The EM algorithm [3] effectively handles the non-linearity introduced by quantization by treating the unquantized signal as a latent variable. This iterative approach allows for maximum likelihood estimation in scenarios where closed-form solutions are unavailable. The resulting GLRT detector is adaptive to unknown signal and noise statistics, making it highly suitable for practical communication systems where these parameters may fluctuate [4], [5].

VI. CONCLUSION

This paper presented an adaptive detection framework for Gaussian signals with jointly unknown statistical parameters in low-resolution quantized systems. By employing the Expectation-Maximization (EM) algorithm, we derived iterative procedures for estimating signal and noise variances from quantized observations. The EM-based Generalized Likelihood Ratio Test (GLRT) detector provides a robust solution for challenging detection scenarios prevalent in modern communication systems. Future work will involve extensive simulations to verify the theoretical derivations and compare the detector's performance with existing methods and theoretical limits, with a particular focus on the impact of different quantization levels and signal-to-noise ratios.

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