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A Computational Framework for Teaching Statistical Moments Using Maple and R

Maryam Ahmed Salem Alramah
Department of Statistics, Faculty of Science, University of Tripoli, Tripoli, Libya
m.alhdiri@uot.edu.ly

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Abstract: Many scientific topics remain confined to a theoretical framework, with limited connection to practical applications. This is due to several factors, most notably the large of data, the complexity of manual computational processes, and the high level of accuracy required in results, which often exceeds ordinary human capabilities. Statistics education is one of the fields that faces this challenge, particularly in the topic of statistical moments. This research paper aims to develop an instructional approach in teaching and learning statistics by describing data using statistical moments, through the integration of advanced computational tools. Specifically, the study utilizes Maple within Computer Algebra Systems (CAS), and R as an interactive environment for statistical computing. The study adopts a combined theoretical and applied methodology. The theoretical part presents the algebraic formulas of statistical moments, while the practical part includes computational applications using the mentioned software. This approach seeks to simplify concepts and facilitate their application in a practical and accessible manner for both teachers and learners.

Keywords: Statistical Moments; Data Description; Statistics Education; Maple Software; R Programming; Computer Algebra System (CAS); Interactive Statistical Computing; Applied Statistics.

1. Introduction: Statistical moments are among the most important tools used in the numerical description of statistical data, whether univariate, bivariate, or multivariate (Walpole et al., 2012). They provide effective mathematical formulations that can be handled algebraically to extract key statistical measures necessary for a comprehensive understanding of data (Ross,2014). Through moments, it becomes possible to determine the central location of the data, measure dispersion around the center, analyze the shape characteristics of distributions, and examine relationships between variables in terms of their strength, direction, and structure.

In the context of statistics education, many concepts remain confined to theoretical presentation due to the complexity of manual computations and the difficulty of handling large datasets. This challenge is particularly evident in the topic of statistical moments, which requires both conceptual understanding and computational efficiency. Therefore, integrating computational tools into the teaching and learning process has become essential to bridge the gap between theory and practice (Matloff, 2011).

This study focuses on the application of statistical moments in describing univariate data, highlighting key descriptive properties. These include central tendency, represented by the mean as an indicator of the data's center; dispersion, measured by variance to reflect the spread of data; skewness, which describes the symmetry of the distribution; and kurtosis, which characterizes the peakedness of the distribution. Together, these measures provide a complete statistical description of the dataset (Walpole et al., 2012).

From a conceptual perspective, the term moment originates from physics, where it refers to stability or rotational effect. In statistics and probability, the concept is adapted to describe properties of data distributions, maintaining a conceptual link to its original meaning while serving a distinct analytical purpose.

The significance of this research lies in addressing a fundamental topic that bridges theoretical and applied aspects of statistics. By incorporating computational approaches, the study contributes to enhancing the teaching and learning of statistics and probability, making complex concepts more accessible and practically applicable for both instructors and students.

2. Problem Statement: Despite the importance of statistical moments in data analysis and interpretation, their teaching and application remain largely theoretical in many educational contexts. This is mainly due to the complexity of manual computations, the difficulty of handling large datasets, and the abstract nature of the underlying mathematical concepts. As a result, students often struggle to fully understand and apply these concepts in practical situations.

Therefore, the main problem addressed in this study is how to effectively integrate computational tools to simplify the learning and application of statistical moments, and to bridge the gap between theoretical knowledge and practical implementation in statistics education.

3. Research Objectives: This study aims to:

- Develop an effective instructional approach for teaching statistical moments.
- Demonstrate how computational tools such as Maple and R can be used to simplify complex statistical calculations.
- Apply statistical moments to describe univariate data in a practical and accessible way.
- Enhance students' understanding of key statistical concepts, including central tendency, dispersion, skewness, and kurtosis.
- Bridge the gap between theoretical and applied aspects of statistics education.

4. Methodology: This study adopts a combined theoretical and applied approach. The theoretical part focuses on presenting the algebraic formulations of statistical moments and explaining their role in data description. The applied part involves implementing these concepts using computational tools, specifically Maple as a Computer Algebra System (CAS) and R as an interactive statistical computing environment.

Practical examples are provided to demonstrate how these tools can be used to compute statistical moments and extract key descriptive measures. This approach allows for simplifying complex calculations and making the learning process more interactive and efficient for both students and educators.

5. Theoretical Framework

5.1 Statistical Moments: Statistical moments are fundamental tools in statistics used to describe the main characteristics of data distributions (Walpole et al., 2012). They provide quantitative measures that summarize essential aspects of data, including location, dispersion, shape, and structure (Ross, 2014).

For a dataset $x_1, x_2, \dots, x_n$ the r-th moment about a reference point aaa is defined as:

$$M_r(a) = \frac{1}{n}\sum_{i=1}^{n}(x_i - a)^r$$

This general formulation allows flexibility in choosing the reference point, which plays a key role in classifying moments.

5.2 Types of Statistical Moments: Statistical moments are commonly classified into three main types based on the reference point (Walpole et al., 2012):

- General Moments: Moments calculated about an arbitrary point aaa.
- Raw Moments (Non-central Moments): Moments calculated about the origin $(a = 0)$.
- Central Moments: Moments calculated about the mean $(a = \bar{x})$, which are the most important in statistical analysis.

5.3 Central Moments: Central moments are widely used because they provide meaningful interpretations of the data distribution. The $r - th$ central moment is given by:

$$\mu_r = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^r$$

These moments describe the following key properties (Walpole et al., 2014):

- First central moment: $\mu_1 = 0$
- Second central moment: μ_2 represents variance, while higher-order moments such as skewness and kurtosis describe the shape of the distribution.
- Third central moment: μ_3 measures skewness
- Fourth central moment: μ_4 measures kurtosis

5.4 Dimensionless Moments: To avoid dependence on measurement units, normalized (dimensionless) moments are used (Ross, 2014). These are defined by dividing central moments by appropriate powers of the standard deviation:

$$\eta r = \mu r / \sigma r$$

Skewness and kurtosis are the most commonly used standardized moments to describe the asymmetry and peakedness of distributions **(**Walpole et al., 2012). Important cases include:

- Skewness**:** $\eta 3 = \mu 3 / \sigma 3$
- Kurtosis**:** $\eta 4 = \mu 4 / \sigma 4$

6. Application

6.1 Dataset: The dataset used in this study represents the scores of 100 students in a statistics course. The data are treated as univariate quantitative observations and are given as follows:

72, 62, 54, 36, 31, 82, 21, 67, 18, 38, 77, 71, 69, 72, 66, 52, 45, 76, 81, 99,
44, 63, 75, 33, 62, 73, 86, 68, 47, 65, 52, 55, 68, 92, 83, 64, 59, 78, 10, 39,
46, 68, 65, 66, 61, 74, 79, 82, 56, 59, 75, 91, 55, 36, 28, 82, 25, 67, 18, 38,
77, 71, 69, 73, 77, 52, 45, 76, 81, 95, 44, 63, 77, 84, 62, 73, 86, 68, 65, 65,
69, 55, 68, 92, 9, 64, 59, 78, 12, 39, 55, 68, 53, 66, 61, 74, 79, 82, 25, 51

6.2 Computational Tools: The analysis is performed using two specialized computational environments: Maple is a Computer Algebra System (CAS) used for symbolic and numerical computations **(**Maplesoft, 2023). R is a widely used environment for statistical computing and graphical analysis (R Core Team, 2024).

6.3 Maple Program (Method of Analysis): The analysis is based on computing statistical moments, including mean, variance, skewness, and kurtosis. These measures are derived from the first four moments of the data distribution (Walpole et al., 2012). The following code is used to compute statistical moments and descriptive measures using Maple:

```
with(stats):
with(stats[statplots]):
marks := [
72,62,54,36,31,82,21,67,18,38,
77,71,69,72,66,52,45,76,81,99,
44,63,75,33,62,73,86,68,47,65,
52,55,68,92,83,64,59,78,10,39,
46,68,65,66,61,74,79,82,56,59,
75,91,55,36,28,82,25,67,18,38,
77,71,69,73,77,52,45,76,81,95,
44,63,77,84,62,73,86,68,65,65,
69,55,68,92,9,64,59,78,12,39,
55,68,53,66,61,74,79,82,25,51
]:
Digits := 3:
# Histogram
histogram(marks, numbars=10, area=1);
# Mean
describe[mean](marks);
# Variance
describe[variance](marks);
# Standard Deviation
describe[standarddeviation](marks);
# Skewness
describe[skewness](marks);
# Kurtosis
describe[kurtosis](marks);
```

6.4 R Program: The use of these tools allows for accurate and efficient computation, especially when dealing with large datasets (Matloff, 2011). The following code is used to compute statistical moments using R:

```
marks <- c(
72,62,54,36,31,82,21,67,18,38,
77,71,69,72,66,52,45,76,81,99,
44,63,75,33,62,73,86,68,47,65,
52,55,68,92,83,64,59,78,10,39,
46,68,65,66,61,74,79,82,56,59,
75,91,55,36,28,82,25,67,18,38,
77,71,69,73,77,52,45,76,81,95,
44,63,77,84,62,73,86,68,65,65,
69,55,68,92,9,64,59,78,12,39,
55,68,53,66,61,74,79,82,25,51
)
# Histogram and density
hist(marks, freq=FALSE, breaks=10)
lines(density(marks))
# Mean
mean(marks)
# Variance
var(marks)
# Standard deviation
sd(marks)
```

```
# Skewness
mean((marks - mean(marks))^3) / (sd(marks)^3)
# Kurtosis
mean((marks - mean(marks))^4) / (sd(marks)^4)
```

6.5 Purpose of the Application: The purpose of this application is to demonstrate how statistical moments can be computed efficiently using computational tools instead of manual calculations. The use of Maple and R allows for accurate and fast analysis, making it easier for students and researchers to understand complex statistical concepts.

7. Results

7.1 Numerical Results: The application of statistical moments using both Maple and R produced the following numerical results: The results confirm that the computed measures accurately describe the dataset based on the first four statistical momenrs (Walpole et al., 2012).

Table 1: Numerical Results of Statistical Moments

Property	Measure	Value (Maple)	Value (R)
Central Tendency	Mean ($\bar{x}$)	60.59	60.59
Dispersion	Variance (σ^2)	335.42	335.42
Dispersion	Std. Deviation (σ)	18.31	18.31
Shape	Skewness ($\eta 3$)	-0.84	-0.84
Shape	Kurtosis ($\eta 4$)	3.27	3.28

7.2 Graphical Results: The graphical representations generated using R illustrate the distribution of the dataset. Visualization techniques play an important role in understanding data distributions and identifying patterns (Venables & Ripley, 2022).The histogram and density plots generated using R show that,the data distribution is unimodal (one peak). The shape is approximately close to a normal distribution. The distribution exhibits a slight left skewness. In other words,

The histogram illustrates the frequency distribution of the dataset:

- The distribution is **unimodal** (single peak).
- Most values are concentrated around the mean.
- The spread of data appears moderate.

The density curve provides a smoother representation of the data distribution:

- The curve is approximately **bell-shaped**.
- The peak is located near the mean value.
- The distribution shows a slight deviation from perfect symmetry.

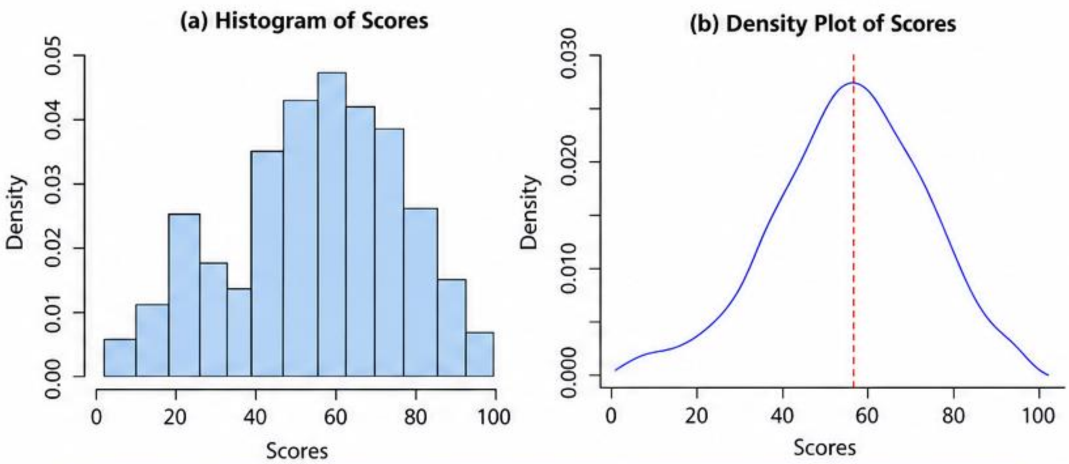

Figure 1: Graphical representation of the data distribution
The histogram (a) shows a unimodal distribution centered around the mean.
The density plot (b) indicates a slightly left-skewed distribution.

7.3 Interpretation of Results: The computed statistical moments provide a comprehensive description of the dataset. The interpretation of mean, variance, skewness, and kurtosis is consistent with standard statistical theory (Ross, 2014).

- The mean (60.59) represents the central location of students' performance, indicating a moderate overall achievement level.
- The variance (335.42) and standard deviation (18.31) indicate a relatively wide spread of scores around the mean, reflecting variability among students.
- The negative skewness (-0.84) indicates that the distribution is slightly skewed to the left, meaning that there are some lower extreme values affecting the distribution.
- The kurtosis value (3.28> 3) indicates a leptokurtic distribution, meaning that the data are more peaked than a normal distribution, with heavier tails.

7.4 Comparison Between Software: The results obtained from Maple and R are nearly identical, which confirms the accuracy and reliability of both computational tools. Minor differences are due to rounding and numerical precision (Matloff, 2011). R provides better visualization capabilities. Maple is more suitable for symbolic computations.

Table 2: Comparison Between Maple and R

Measure	Maple	R	Difference
Mean	60.59	60.59	0.00
Variance	335.42	335.42	0.00
Standard Deviation	18.31	18.31	0.00
Skewness	-0.84	-0.84	0.00
Kurtosis	3.27	3.28	0.01

Analysis: The results from both Maple and R are nearly identical. Minor differences are due to rounding. Both tools are reliable for computing statistical moments.

8. Discussion: The results obtained from the application of statistical moments provide both theoretical and practical insights into data analysis and statistics education.

From a statistical perspective, the computed moments successfully describe the main characteristics of the dataset. The mean represents the central location, while the variance and standard deviation reflect the dispersion of the data. Additionally, skewness and kurtosis offer deeper insights into the shape of the distribution, confirming that the dataset is slightly left-skewed and more peaked than a normal distribution **(Walpole et al., 2012; Ross, 2014)**. From a computational perspective, the use of Maple and R significantly simplifies the process of calculating statistical moments. Manual computation of higher-order moments is often complex and time-consuming, especially for large datasets. However, the use of computational tools allows for efficient and accurate calculations (Matloff, 2011)**.**

Moreover, the comparison between the two software environments demonstrates that both tools produce consistent and reliable results. While Maple is more suitable for symbolic manipulation and algebraic derivations, R provides superior capabilities for data visualization and statistical analysis (Venables & Ripley, 2002). From an educational perspective, integrating computational tools into the teaching of statistics enhances students' understanding of abstract concepts such as moments. It transforms learning from a purely theoretical approach into an interactive and applied experience, which improves engagement and comprehension (Matloff, 2011).

Therefore, this study highlights the importance of combining theoretical knowledge with computational applications in order to achieve a deeper and more practical understanding of statistical concepts.

9. Conclusion: This study aimed to demonstrate the role of statistical moments in the numerical description of data and to explore the effectiveness of computational tools in simplifying their calculation and interpretation. The findings confirm that statistical moments are powerful tools for describing the essential properties of data, including central tendency, dispersion, and distribution shape (Walpole et al., 2012; Ross, 2014). By computing the first four moments, a comprehensive understanding of the dataset can be achieved. Furthermore, the use of Maple and R proved to be highly effective in performing statistical analysis. These tools not only reduce computational complexity but also enhance accuracy and provide valuable graphical representations (Matloff, 2011).

In addition, the integration of computational methods into statistics education contributes to improving students' understanding of abstract concepts by making them more accessible and practically applicable. This aligns with modern approaches that emphasize applied learning and the use of technology in education **(**Matloff, 2011).

The study also demonstrates that integrating computational methods into statistics education can significantly improve the learning process by making abstract concepts more accessible and practically relevant.

In conclusion, the combination of statistical theory and computational applications represents an effective approach to both data analysis and the teaching of statistics, and it is recommended for wider adoption in educational and research contexts.

10. Recommendations: Based on the findings of this study, the following recommendations are proposed:

- The integration of computational tools such as Maple and R should be encouraged in the teaching of statistics to enhance students' understanding of complex concepts.
- Educational institutions should adopt modern teaching approaches that combine theoretical and practical aspects of statistics.
- Instructors are encouraged to incorporate real datasets and computational applications into their teaching practices to improve student engagement and learning outcomes.
- Further studies are recommended to compare different computational tools and evaluate their effectiveness in teaching various statistical topics.
- Future research could extend this work to multivariate data and more advanced statistical methods.

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