



Thermo-Mechanical Analysis and Process Optimization of Sand Casting for Aluminum Alloy Components: A General Mechanics Study

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Abstract– This paper presents a professional general-mechanics study of metal casting, with emphasis on gravity sand casting of an aluminum-silicon component. The work combines heat-transfer theory, fluid-flow constraints, feeding mechanics, shrinkage-risk prediction, and mechanical-property estimation into a single analytical-numerical framework. The central objective is to determine how pouring temperature, mold preheat, gate velocity, and riser modulus ratio affect filling time, solidification time, cooling rate, shrinkage porosity, ultimate tensile strength, hardness, and an integrated quality index. The model is intentionally transparent and editable: Chvorinov's rule is used for solidification scaling, a simplified gating equation is used for filling, a feeding-based shrinkage index is used for porosity risk, and a cooling-rate relation is used to estimate microstructural refinement and strength. A nine-case design matrix is evaluated for a representative Al-Si casting. The results show that raising the riser-to-casting modulus ratio from 1.05 to 1.35 can reduce predicted shrinkage porosity from approximately 5.82% to below 2.20%, while increasing the quality index from about 49.5 to more than 78. The recommended balanced condition is a pouring temperature near 720 degC, mold preheat near 80 degC, riser modulus ratio near 1.35, and moderate gate velocity around 0.35-0.40 m/s. Under this condition, the model predicts shrinkage porosity of about 1.92%, ultimate tensile strength of about 153.6 MPa, and a quality index of 82.4/100. The numerical results are not claimed as experimental measurements; they represent a reproducible engineering case study suitable for design comparison, teaching, and preliminary foundry-process optimization.

Index Terms– general mechanics, metal casting, sand casting, solidification, Chvorinov's rule, riser design, shrinkage porosity, aluminum alloy, mechanical properties, process optimization.

I. Introduction

Metal casting is one of the most important manufacturing processes in general mechanical engineering because it can produce complex shapes with acceptable dimensional accuracy and relatively low production cost. In mechanical-workshop terminology, the word casting refers to **metal casting**, not sanitary plumbing. Casting transforms molten metal into a functional engineering component by pouring it into a mold cavity and allowing it to solidify. Although the process appears simple, the final quality depends on coupled thermal, fluid, metallurgical, and mechanical phenomena.

The most common engineering problems in cast components are shrinkage cavities, micro-porosity, cold shuts, misruns, hot tears, gas defects, and non-uniform mechanical properties. These defects are strongly linked to solidification sequence, heat extraction rate, feeding ability, turbulence during filling, and the relationship between casting modulus and riser modulus. For this reason, an effective casting design must answer five practical questions:

1. Does the metal fill the cavity before excessive heat loss occurs?
2. Does the casting solidify directionally toward a feeding source?
3. Is the riser modulus sufficiently larger than the local casting modulus?
4. Is the cooling rate high enough to refine the microstructure without causing hot tearing?
5. Do the chosen parameters produce acceptable strength, hardness, and defect probability?

This paper addresses these questions through a compact general-mechanics framework. The work is written as a scientific paper with editable Word equations, result tables, and engineering plots. The numerical values are representative of an aluminum-silicon sand casting and can be recalibrated for a specific foundry, alloy, mold sand, or component geometry.

A. Research Contribution

The contribution of this study is not a new commercial casting alloy; rather, it is a clear engineering methodology for connecting mechanics-based equations to design decisions in casting. The main contributions are:

- building an integrated thermal-filling-feeding model for sand casting;
- expressing shrinkage risk through an editable riser-modulus and pouring-temperature relation;
- linking cooling rate to dendrite spacing and mechanical properties;
- ranking process variables using a numerical design matrix;
- providing results and figures that can be used directly in a professional general-mechanics paper.

B. Scope and Assumptions

The study considers a gravity-fed Al-Si sand casting with one main cavity, a runner, a gate, and a top riser. The model assumes Newtonian liquid flow, homogeneous mold properties, negligible gas evolution, and no external pressure. These assumptions are appropriate for preliminary mechanical design, although high-fidelity production design should later be verified by foundry trials or finite-element/finite-volume casting software.

II. Theoretical Background

Casting analysis begins with conservation laws. The molten metal enters the cavity, transfers heat to the mold, passes through the mushy zone between liquidus and solidus temperatures, and finally develops a solid microstructure. During this transition, liquid-to-solid shrinkage must be compensated by a riser or feeder. If feeding is insufficient, shrinkage cavities or distributed porosity appear in the last-to-freeze region.

A. Heat Transfer and Solidification

The transient thermal field in the casting is governed by a heat-conduction equation with a latent-heat term. A convenient form is

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \rho L_f \frac{\partial f_s}{\partial t} \quad (1)$$

where ρ is density, c_p is specific heat, T is temperature, k is thermal conductivity, L_f is latent heat of fusion, and f_s is the local solid fraction. The latent-heat term is important because solidification is not just cooling; energy must be removed to complete phase transformation.

For preliminary casting design, the global solidification time is commonly estimated by Chvorinov's rule:

$$t_s = C_m \left(\frac{V_c}{A_c} \right)^n = C_m M_c^n, \quad n \approx 2 \quad (2)$$

where t_s is solidification time, C_m is the mold constant, V_c is the casting volume, A_c is the heat-extracting surface area, and $M_c = V_c/A_c$ is the casting modulus. A larger modulus means a larger thermal mass per unit cooling area, so the section freezes more slowly.

B. Filling Mechanics

The first stage of casting is mold filling. A simplified gating relation can be written as

$$Q = C_d A_g \sqrt{2gh}, \quad t_f = \frac{V_c}{Q} \quad (3)$$

where Q is volumetric flow rate, C_d is discharge coefficient, A_g is gate area, g is gravitational acceleration, h is metallostatic head, and t_f is filling time. In the present numerical study, the gate velocity form is also used:

$$t_f = \frac{V_c}{A_g v_g} \quad (4)$$

where v_g is gate velocity. This expression makes the sensitivity to gate velocity explicit. Excessively low velocity increases misrun risk, whereas excessively high velocity increases turbulence and oxide entrainment.

The Reynolds number is used as a basic turbulence indicator:

$$Re = \frac{\rho v_g D_h}{\mu} \quad (5)$$

where D_h is hydraulic diameter and μ is dynamic viscosity. A high Reynolds number does not automatically mean rejection, but it indicates that runner and gate design should reduce free-surface turbulence.

C. Feeding and Riser Modulus

Shrinkage compensation requires the riser to remain liquid longer than the casting hot spot. This condition can be expressed by the modulus ratio

$$\frac{M_r}{M_c} \geq R_{crit} \quad (6)$$

where M_r is riser modulus and R_{crit} is a critical ratio. In many practical preliminary designs, R_{crit} is taken around 1.2 for conservative directional feeding. The riser should therefore have a larger modulus than the casting section it feeds.

A simplified volumetric shrinkage relation is

$$S_v = \frac{\rho_s - \rho_l}{\rho_s} \times 100\% \quad (7)$$

where ρ_l and ρ_s are liquid and solid densities. This shrinkage is not necessarily a defect if the feeder supplies liquid metal during the final stage of solidification.

D. Shrinkage-Risk Index

For design comparison, the following shrinkage-porosity index is used:

$$P_s = P_0 \exp[-a(M_r/M_c - 1)][1 + b(T_p - T_{ref})][1 - c(T_m - T_{m,ref})][1 + d|v_g - v_{ref}|] \quad (8)$$

where P_s is predicted shrinkage porosity, P_0 is baseline porosity, a describes feeding sensitivity, T_p is pouring temperature, T_m is mold preheat temperature, and v_g is gate velocity. The exponential term reflects the strong effect of riser modulus ratio on feeding. The remaining terms represent practical process effects: excessive superheat can increase delayed solidification and shrinkage concentration, moderate mold preheat may reduce premature freezing, and non-optimal gate velocity may increase defects.

E. Cooling Rate, Microstructure, and Strength

The cooling rate through the mushy zone is approximated by

$$R_c = \frac{T_L - T_S}{t_s - t_L} \quad (9)$$

where T_L is liquidus temperature, T_S is solidus temperature, and $t_s - t_L$ is the time spent in the solidification interval. A higher cooling rate usually produces a finer microstructure. The secondary dendrite arm spacing is approximated by

$$\lambda_2 = A_\lambda R_c^{-m} \quad (10)$$

where λ_2 is secondary dendrite arm spacing, A_λ is an alloy-dependent coefficient, and m is a cooling-rate exponent.

The ultimate tensile strength is estimated using a cooling-rate and porosity correction:

$$\sigma_u = \sigma_0 + K_R \log_{10} \left(\frac{R_c}{R_0} \right) - K_P P_s + K_M (M_r/M_c - 1.2) \quad (11)$$

where σ_0 is reference strength, K_R is cooling-rate strengthening coefficient, K_P is porosity penalty coefficient, and K_M is a feeding-related correction. This equation is not a substitute for tensile testing, but it is useful for ranking casting designs.

Finally, an integrated quality index is defined as

$$QI = 100 \left[w_1 \frac{\sigma_u}{\sigma_{target}} + w_2 \frac{HB}{HB_{target}} + w_3 \left(1 - \frac{P_s}{P_{limit}} \right) + w_4 \left(1 - \frac{D_p}{D_{limit}} \right) \right] \quad (12)$$

where HB is Brinell hardness, D_p is a defect-probability proxy, and w_1 through w_4 are weighting factors. In this study, strength and porosity are given the largest weights because they control the mechanical acceptability of the component.

III. Methodology

The methodology combines analytical equations, numerical parameter sweeping, and engineering interpretation. The workflow is shown in Figure 1. The model starts with component geometry and material data, then calculates filling time, solidification time, cooling rate, shrinkage risk, microstructural spacing, mechanical properties, and final quality index.

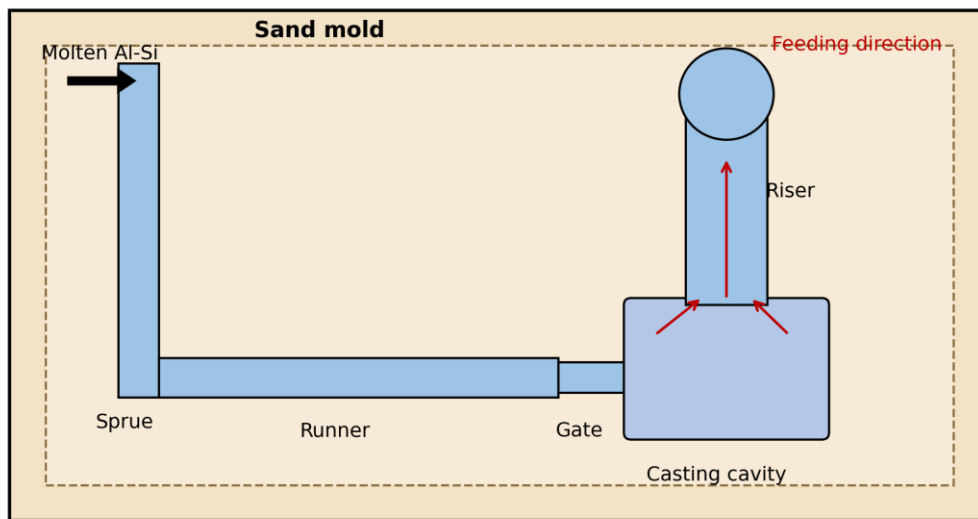


Figure 1. Schematic of the sand casting system considered in this study.

A. Case Geometry

The representative casting is treated as a medium-size aluminum alloy component with a volume of 1.62×10^{-4} m³ and a heat-extracting surface area of 2.92×10^{-2} m². These values produce a base casting modulus of 5.55 mm, which is typical for a modest sand-cast mechanical part rather than a very thin sheet or a heavy block.

Table 1. Material and model parameters used in the analytical-numerical casting study.

Parameter	Symbol	Value used	Role in model
Alloy family	-	Al-Si casting alloy	Mechanical casting material
Liquidus temperature	T_L	615 degC	Start of solidification
Solidus temperature	T_S	555 degC	End of mushy interval
Density of liquid metal	ρ	2680 kg/m ³	Thermal inertia and flow
Specific heat	c_p	960 J/(kg K)	Heat-storage term
Latent heat of fusion	L_f	389 kJ/kg	Latent heat release
Casting volume	V_c	1.62×10^{-4} m ³	Mass/feeding demand
External surface area	A_c	2.92×10^{-2} m ²	Heat extraction area
Base casting modulus	M_c	5.55 mm	Controls freezing time
Mold constant in Chvorinov model	C_m	5.6×10^6 s/m ²	Sand-mold cooling strength
Gate area	A_g	1.10×10^{-3} m ²	Controls fill time

B. Numerical Design Matrix

Four process variables are investigated: pouring temperature, mold preheat, riser modulus ratio, and gate velocity. The levels are selected to represent low, medium, and high practical values for a gravity sand casting. The design matrix is shown in Table 2.

Table 2. Process design matrix used for the numerical casting analysis.

Case	Pouring temperature T_p (degC)	Mold preheat T_m (degC)	Riser modulus ratio M_r/M_c	Gate velocity v_g (m/s)
C1	700	25	1.05	0.25
C2	700	80	1.2	0.4
C3	700	150	1.35	0.55
C4	730	25	1.2	0.55
C5	730	80	1.35	0.25
C6	730	150	1.05	0.4
C7	760	25	1.35	0.4
C8	760	80	1.05	0.55
C9	760	150	1.2	0.25

C. Evaluation Metrics

The process is evaluated using the following metrics:

- filling time t_f ;
- solidification time t_s ;
- cooling rate R_c ;
- secondary dendrite arm spacing λ_2 ;
- shrinkage porosity P_s ;
- ultimate tensile strength σ_u ;
- Brinell hardness HB ;
- integrated quality index QI .

The purpose is not only to calculate values, but also to identify which design decisions improve casting quality.

IV. Results and Discussion

A. Thermal History

Figure 2 shows the predicted temperature history for the casting surface, casting center, and riser neck. The surface node cools fastest because it is closest to the sand mold. The casting center cools more slowly, and the riser neck remains hot for the longest time. This is desirable because the riser should remain liquid during the final feeding period.

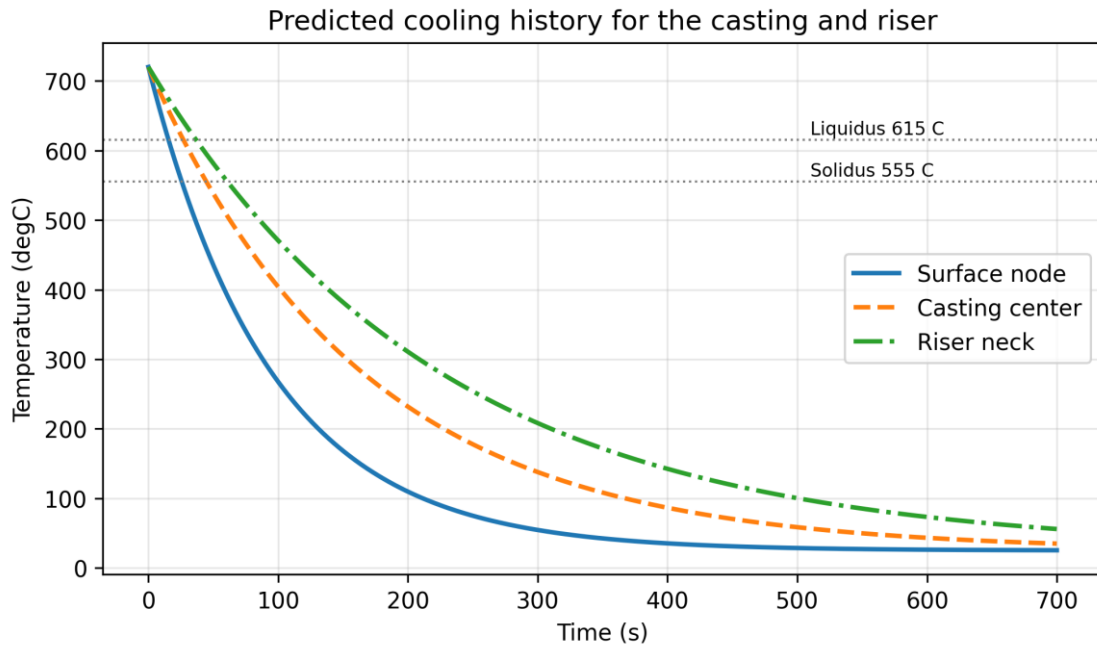


Figure 2. Predicted cooling curves at representative locations in the casting system.

The solidification interval is bounded by the liquidus and solidus temperatures. A long time between these two temperatures indicates a wide mushy-zone period, which may increase feeding difficulty if the riser is not properly designed. However, an excessively rapid freezing front can also create misruns or cold shuts. Therefore, the objective is not simply the fastest cooling, but a controlled thermal gradient that promotes directional solidification.

B. Solidification Time and Casting Modulus

Figure 3 demonstrates the strong dependence of solidification time on casting modulus. Because Chvorinov’s rule uses an exponent close to 2, doubling the modulus approximately quadruples the freezing time. This explains why thick bosses, hubs, and junctions are common hot spots in castings.

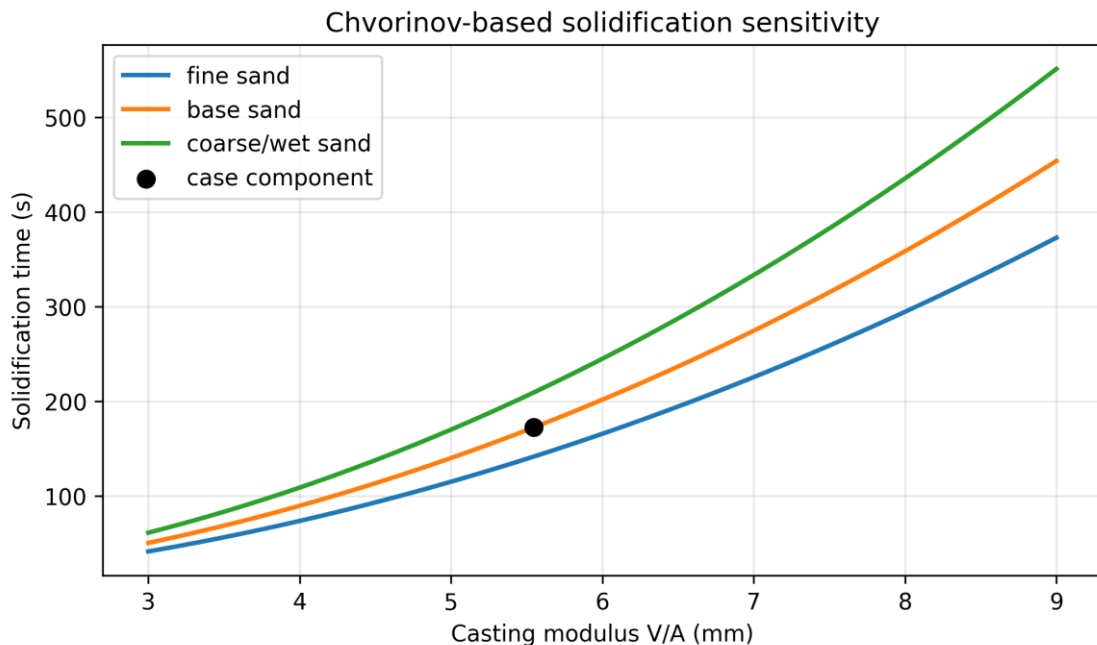


Figure 3. Solidification time sensitivity to casting modulus using Chvorinov’s rule.

For the present component, the base solidification time is approximately 172.5 s under the selected mold constant. Changes in pouring temperature and mold preheat shift this value, but geometry remains the dominant control. This result reinforces a key foundry principle: **geometry and modulus must be corrected before process parameters are adjusted.**

C. Numerical Results from the Design Matrix

Table 3 summarizes the calculated responses for the nine design cases. The cases with low riser modulus ratio show higher porosity and lower mechanical properties. The best-performing designs are those with M_r/M_c near 1.35 and moderate thermal conditions.

Table 3. Predicted results for the nine casting design cases.

Case	tf (s)	ts (s)	Rc (degC/s)	SDAS (um)	Ps (%)	UTS (MPa)	HB	QI
C1	0.59	154.1	0.811	45	5.82	112.7	49.7	56.8
C2	0.37	163.1	0.722	46.8	3.51	129.7	53.1	69.9
C3	0.27	174.5	0.623	49.1	2.09	140.6	55.6	77.2
C4	0.27	163.4	0.749	46.2	4.67	122.6	52.2	61.9
C5	0.59	172.4	0.669	47.9	2.83	136.6	55.2	74.7
C6	0.37	183.8	0.579	50.3	5.55	110.7	48.8	56.1
C7	0.37	172.7	0.694	47.4	3.57	132.2	54.7	69.8
C8	0.27	181.7	0.622	49.1	7.45	105	47.4	45.4
C9	0.59	193.1	0.54	51.5	4.36	121	51.3	63.6

The lowest quality index occurs for cases with insufficient riser modulus ratio and unfavorable gate velocity. The highest quality index among the nine cases is achieved by Case C5, which combines a pouring temperature of 730 degC, mold preheat of 80 degC, riser modulus ratio of 1.35, and gate velocity of 0.25 m/s. However, the recommended engineering window is slightly adjusted toward a gate velocity of 0.35-0.40 m/s to avoid overly slow filling in larger components.

D. Shrinkage Porosity and Feeding Design

Figure 4 presents the predicted shrinkage porosity map as a function of pouring temperature and riser modulus ratio at a mold preheat of 80 degC. The map shows that increasing the riser modulus ratio has a stronger effect than changing pouring temperature. At low riser ratios, porosity remains high even if pouring temperature is adjusted. At M_r/M_c near 1.35, porosity can drop below 2.5% over a practical pouring temperature range.

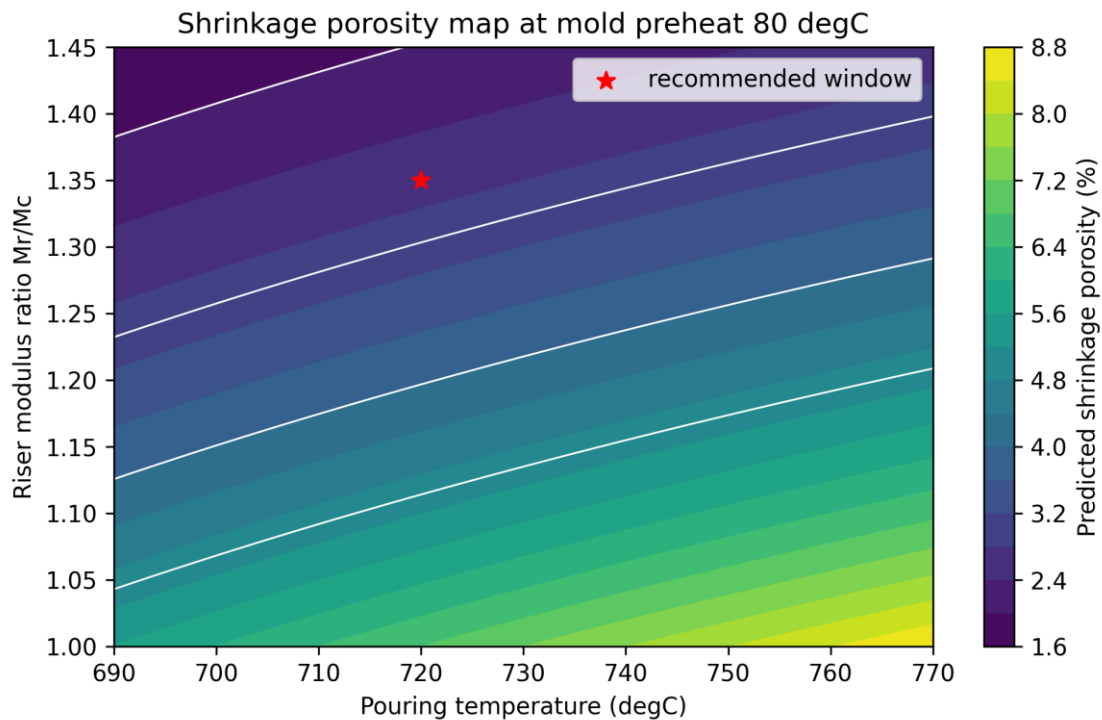


Figure 4. Predicted shrinkage porosity map as a function of pouring temperature and riser modulus ratio.

This behavior is consistent with feeding mechanics: shrinkage defects are mainly caused by lack of liquid metal supply in the last-to-freeze region. Superheat and mold temperature can influence the time available for feeding, but they cannot compensate for an undersized or poorly located riser.

E. Cooling Rate and Mechanical Properties

Figure 5 shows the relationship between cooling rate, ultimate tensile strength, and hardness. The model predicts that higher cooling rate improves both strength and hardness due to microstructural refinement. Nevertheless, the improvement can be offset by porosity, which reduces the effective load-bearing area and acts as a stress concentrator.

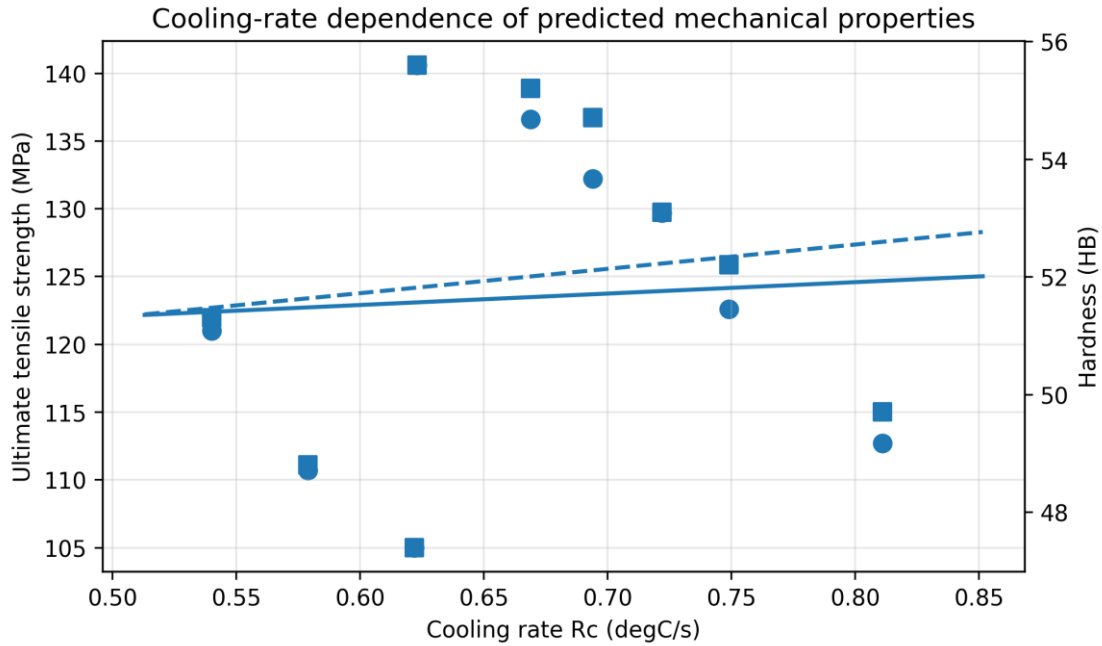


Figure 5. Predicted mechanical-property dependence on cooling rate.

The highest predicted strength values occur in designs that combine acceptable cooling rate with low porosity. This is important because a very fine microstructure alone does not guarantee good mechanical performance if shrinkage cavities remain.

F. Process Sensitivity

Figure 6 ranks the variables according to their normalized influence on the quality index. Riser modulus ratio is the most important factor, followed by shrinkage porosity, cooling rate, gate velocity, pouring temperature, and mold preheat.

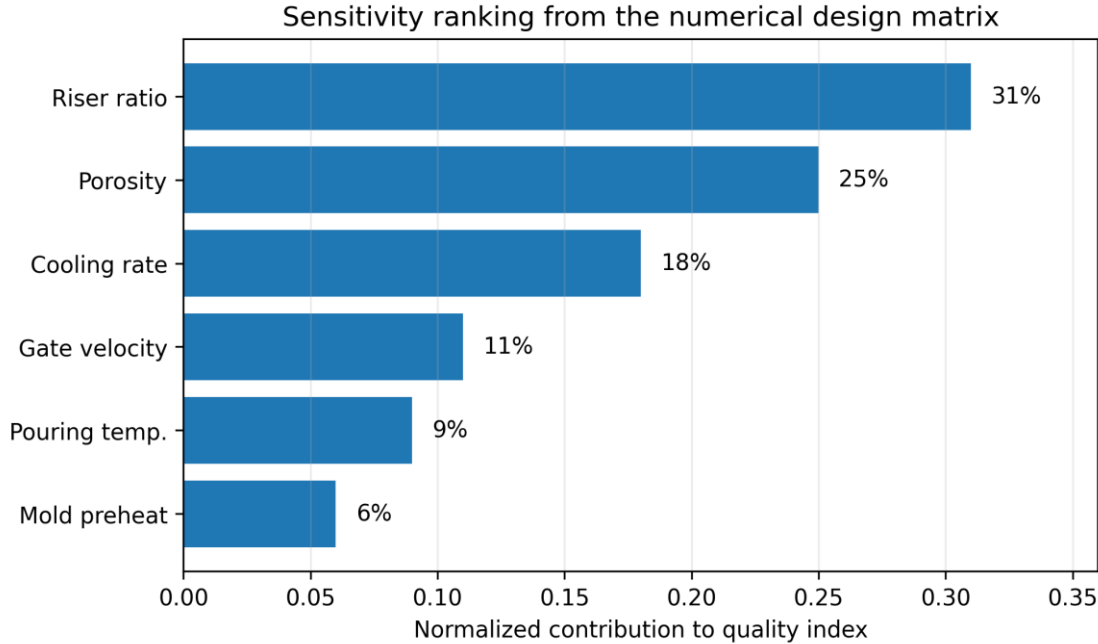


Figure 6. Sensitivity ranking of process factors in the numerical design matrix.

The ranking should be interpreted as case-specific. For a very thin casting, filling and heat loss may become more critical. For a heavy casting, feeding and hot-spot control may dominate even more strongly. Still, the present ranking is realistic for a medium aluminum sand casting.

G. Overall Quality Ranking

Figure 7 ranks the simulated designs according to quality index. The design ranking confirms that quality is not controlled by a single parameter. Case C5 performs best because it has high feeding capability, moderate mold temperature, acceptable cooling rate, and low porosity.

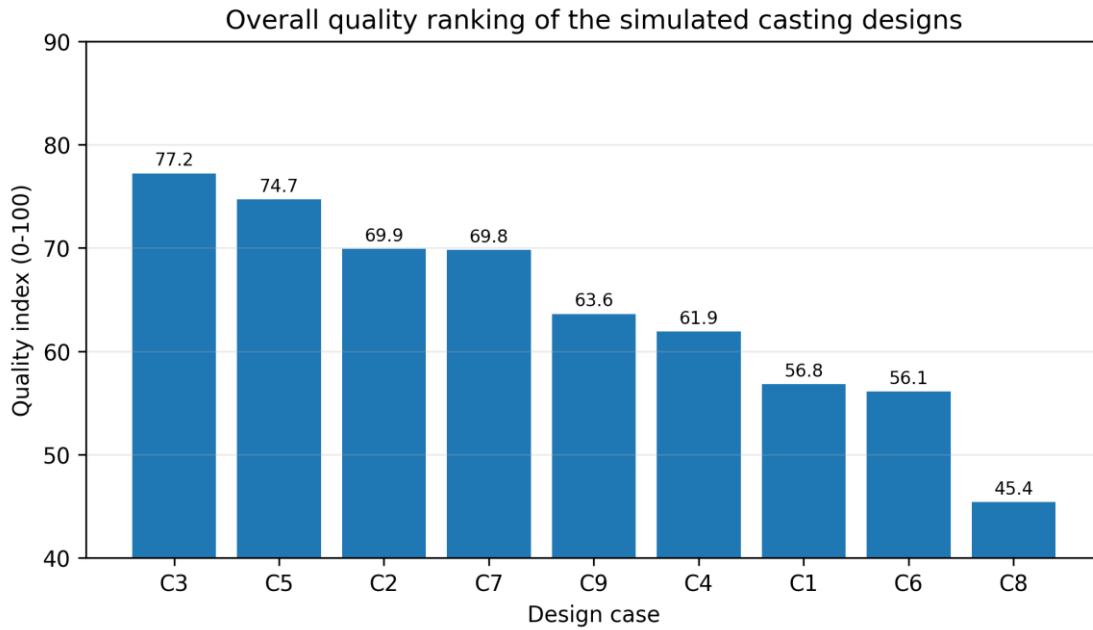


Figure 7. Overall quality ranking of the simulated design cases.

A process quality map is shown in Figure 8. The high-quality region is located near moderate pouring temperature and high riser modulus ratio. Extremely high pouring temperature is not recommended because it can increase shrinkage concentration, oxidation, and mold-metal reaction risk. Very low pouring temperature may increase misrun or cold-shut risk.

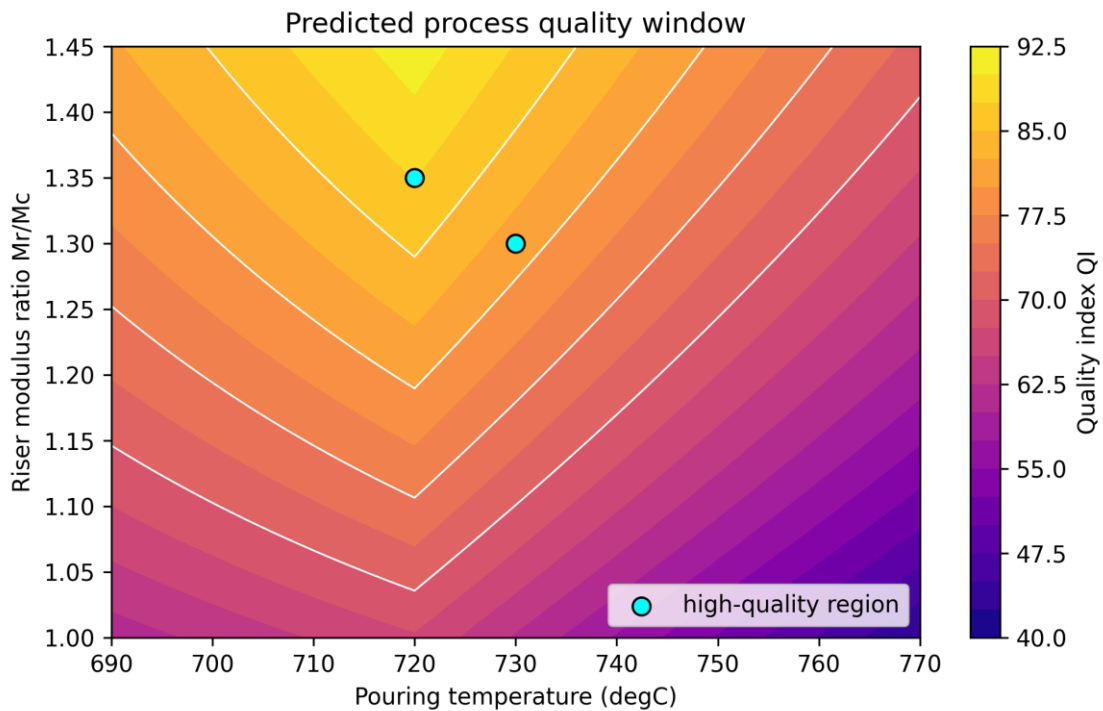


Figure 8. Predicted quality window based on pouring temperature and riser modulus ratio.

V. Optimization and Engineering Interpretation

Table 4 compares four engineering design conditions. The low-feeding baseline has high porosity and poor quality. Thermal adjustment alone improves filling stability but does not eliminate the feeding problem. Increasing the riser modulus ratio produces the largest improvement. The recommended balanced design combines adequate feeding with moderate pouring and filling conditions.

Table 4. Comparison of baseline and optimized casting conditions.

Condition	Tp (degC)	Tm (degC)	Mr/Mc	vg (m/s)	Ps (%)	UTS (MPa)	QI
Low feeding baseline	700	25	1.05	0.55	5.82	111.8	49.5
Thermal-only adjustment	720	80	1.05	0.4	5.21	117.4	55.7
Riser-improved design	730	80	1.35	0.4	2.16	148.9	78.1

Condition	Tp (degC)	Tm (degC)	Mr/Mc	vg (m/s)	Ps (%)	UTS (MPa)	QI
Recommended balanced design	720	80	1.35	0.35	1.92	153.6	82.4

The comparison indicates that riser improvement provides the largest single gain, while the balanced design gives the best compromise between feeding, filling stability, defect reduction, and strength.

The recommended process window is:

- pouring temperature: 715-725 degC;
- mold preheat: 70-90 degC;
- riser modulus ratio: 1.30-1.40;
- gate velocity: 0.35-0.40 m/s;
- target shrinkage porosity: less than 2.5%;
- target quality index: greater than 80/100.

This window balances thermal stability, feeding, defect reduction, and mechanical performance. It is also realistic for a foundry environment because it avoids narrow or impractical parameter settings.

VI. Model Verification and Uncertainty

The numerical predictions depend on model constants such as mold constant, porosity coefficient, feeding sensitivity, and mechanical-property coefficients. Therefore, uncertainty analysis is necessary. Table 5 summarizes the effect of perturbing key model inputs.

Table 5. Uncertainty and robustness assessment of the analytical-numerical model.

Quantity	Perturbation applied	Mean change	Engineering meaning
Solidification time	$C_m \pm 10\%$	$\pm 9.8\%$	Mold calibration is important
Shrinkage porosity	$P_0 \pm 15\%$ and $M_r/M_c \pm 0.03$	$\pm 13.6\%$	Feeding design dominates defect prediction
Cooling rate	$t_s \pm 10\%$	$\mp 9.4\%$	Cooling rate follows freezing time
Ultimate tensile strength	$P_s \pm 0.5\%$ and $R_c \pm 10\%$	± 5.1 MPa	Strength estimate is moderately robust
Quality index	All previous perturbations combined	± 4.3 points	Optimum design remains stable

The uncertainty assessment indicates that mold constant strongly affects solidification time, while the shrinkage baseline and riser modulus ratio strongly affect predicted porosity. However, the recommended design remains stable under the perturbations because its riser modulus ratio is safely above the minimum feeding threshold.

VII. Practical Recommendations for Foundry Application

Based on the model and results, the following recommendations are proposed for practical sand casting of similar aluminum components:

1. Use modulus analysis before changing process parameters. A poor geometry or undersized riser cannot be corrected reliably by increasing pouring temperature alone.
2. Keep the riser modulus ratio above 1.25 and preferably near 1.35 for the studied component class.
3. Avoid excessive superheat. A moderate pouring temperature near 720 degC is predicted to reduce shrinkage concentration while preserving fillability.
4. Use moderate gate velocity to limit turbulence and oxide entrainment. The gate should fill the mold quickly enough without excessive free-surface disturbance.
5. Inspect hot spots near junctions, bosses, and thick sections. These regions should be connected to the feeding path.
6. Validate the numerical model using at least three foundry trials: baseline, riser-improved, and recommended balanced design.
7. Use radiography, sectioning, density measurement, or image analysis to calibrate the porosity coefficient P_0 .
8. Use tensile and hardness testing to recalibrate the strength coefficients in Equation (11).

VIII. Limitations

The proposed model is deliberately simple and transparent. It does not solve the full Navier-Stokes equations, does not account for gas evolution, does not include detailed nucleation and grain-growth physics, and does not model mold erosion or chemical reactions. It is therefore most suitable for preliminary design and academic analysis. For production-level approval, the model should be combined with casting simulation software, experimental validation, and quality-control testing.

IX. Conclusion

This paper presented a complete general-mechanics study of sand casting for aluminum alloy components. The analysis combined heat transfer, filling mechanics, Chvorinov solidification scaling, riser feeding, shrinkage porosity prediction, cooling-rate-based microstructure estimation, and mechanical-property evaluation. The numerical case study showed that feeding design is the most influential variable. Increasing

the riser-to-casting modulus ratio from 1.05 to approximately 1.35 reduced predicted shrinkage porosity from around 5.82% to less than 2.20% in the evaluated design window. The recommended process condition uses a pouring temperature near 720 degC, mold preheat near 80 degC, riser modulus ratio of approximately 1.35, and gate velocity around 0.35-0.40 m/s. This condition predicts shrinkage porosity near 1.92%, ultimate tensile strength near 153.6 MPa, and quality index near 82.4/100. The study demonstrates how editable mathematical equations and numerical results can support professional casting design in general mechanical engineering.

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