Properties of Closed Sets and Open Sets in Nano Topology

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Received: 3-3-2026; Revised: 8-3-2026; Accepted: 12-3-2026; Published: 12-5-2026

Abstract

This paper investigates the properties of closed and open sets within the context of nano topology, a specialized branch of topology that addresses the distinct challenges and characteristics inherent to spaces at the nanoscale. We provide a thorough examination of the fundamental definitions, essential properties, and implications of these sets, underscoring their significance in a range of applications, particularly in the fields of nanotechnology and materials science.

Keywords: Closed Sets in nano topology, Open Sets in nano topology, nano g-open , nano g-closed

الملخص العربي

تتناول هذه الورقة البحثية خصائص المجموعات المغلقة والمفتوحة في سياق طوبولوجيا النانو، وهو فرع متخصص من الطوبولوجيا يُعنى بالتحديات والخصائص الفريدة التي تميز الفضاءات على المستوى النانوي. نقدم دراسة معمقة للتعريفات الأساسية والخصائص الجوهرية لهذه المجموعات، ونؤكد على أهميتها في مجموعة واسعة من التطبيقات، لا سيما في مجالي تكنولوجيا النانو وعلوم المواد

الكلمات المفتاحية: المجموعات المغلقة في طوبولوجيا النانو، المجموعات المفتوحة في طوبولوجيا النانو، مجموعات نانو g-المفتوحة، مجموعات نانو-g المغلقة

1 . Introduction

Topology, a fundamental discipline of mathematics, focuses on the properties of spaces that remain invariant under continuous transformations. Among its key concepts are open and closed sets, which are vital tools for understanding the structure and behavior of topological spaces. In recent years, the emergence of nano topology has drawn attention to the unique properties of these sets at the nanoscale, where classical topological concepts must be adapted to account for quantum effects and discrete physical structures.

At the nanoscale, the interactions and arrangements of atoms and molecules introduce complexities that challenge traditional topological frameworks. The behavior of closed and open sets in this context not only enriches our theoretical understanding but also has practical implications in diverse fields such as nanotechnology, materials science, and biophysics. For example, open sets may represent the spatial configurations of nanoparticles, while closed sets represent stable configurations that minimize energy states.

This paper explains the properties and relationships of closed and open sets in nano topology , exploring their definitions, key characteristics, and relevance to nanoscale phenomena. By linking abstract mathematical concepts to practical applications, we seek to enhance the understanding of how these topological properties influence the design and functionality of materials at the nanoscale.

2 . Literature Review

Exploring the properties of closed and open clusters in nanoscale topology has gained significant momentum in recent years, driven by advances in nanotechnology and the need to understand complex nanoscale systems. This literature review summarizes recent research findings, highlighting important contributions and emerging topics in the field.

A fundamental understanding of open and closed clusters remains crucial in nanoscale topology. Recent studies, such as those by Narmatha S. et al., have revisited classical definitions while incorporating nanoscale considerations. Their work emphasizes the need to adapt these concepts to account for the unique physical properties observed at the nanoscale, such as quantum confinement and discrete atomic arrangements. [11]

Quantum mechanics plays a pivotal role in the behavior of nanoscale systems, influencing the properties of open and closed clusters. Siham et al. investigated the impact of quantum effects on the stability of closed clusters in nanoparticle assemblies. Their results indicate that interactions at the atomic level lead to emergent behaviors that can be described using topological principles, enriching our understanding of stability in nanomaterials.[9]

The effects of open and closed clusters are of particular importance in the design and functionality of nanomaterials. Recent research by M. El Sayed demonstrated how to manipulate the topological properties of nanodevices to improve their performance in applications such as drug delivery and biosensing. Their study highlights the importance of understanding these properties for developing materials with tailored functions.[4]

Approaches have been developed to explore the properties of closed and open clusters in nanoscale topology. For example, a study by Nitesh Kumar et al. combined concepts from topology and materials science, investigating how the arrangement of atoms within nanostructures affects both topological properties and physical behaviors. This approach underscores the importance of collaboration across disciplines to address complex challenges in nanoscale research.[13]

There are still many gaps in the literature. Many studies focus on theoretical frameworks without sufficient experimental validation. Furthermore, there is a need for comprehensive models that integrate the various factors affecting topological properties in nano systems. Future research should aim to fill these gaps by conducting experimental studies that validate theoretical predictions and exploring the implications of topological properties for emerging technologies.

Published studies on the properties of closed and open groups in nanoscale topology reflect a growing recognition of their importance in understanding nanoscale phenomena. Recent research has provided valuable insights into the interplay between topology and quantum effects, as well as practical applications in nanotechnology. As the field continues to evolve, a more integrated approach that combines theoretical and experimental studies will be essential to advance our understanding of nanoscale topology and its applications.

3. Definitions[4,6]

3.1 Open Sets: An open set in a topological space is defined as a set that, for every point within it, there exists a neighborhood entirely contained in the set. Formally, a subset U of a topological space X is open if satisfies: $\emptyset$ and X are in τ any union of sets in τ is in τ , and any finite intersection of sets in τ is in τ

3.2 Closed Sets : A set $U \subseteq X$ is closed if its complement X / U is an open set in the topology τ

.3.3 Properties of Open Sets

a) If $\{U_i\}_{i\in I}$is any collection of open sets, then the union $\bigcup_{i\in I} U_i$is also an open set. This property emphasizes the flexibility of open sets.

b) The intersection of a finite number of open sets remains an open set. Specifically, if $U_1, U_2, \dots, U_n$are open, then $U_1 \cap U_2 \cap \dots \cap U_n$is open. This property is pivotal in defining local characteristics.

c) For a subset $Y \subseteq X$, a set $U$is open in the subspace topology on Yif there exists an open set $V \in \tau$such that $U = V \cap Y$. This relationship plays a crucial role in the study of topological properties restricted to subsets.

3.4 Properties of Closed Sets

a) The intersection of any collection of closed sets is a closed set. More formally, if $\{U_i\}_{i\in I}$is a collection of closed sets, then the intersection $\bigcap_{i\in I} U_i$is closed.

b) The union of a finite number of closed sets is also a closed set. Specifically, if $U_1, U_2, \dots, U_n$are closed sets, then $U_1 \cup U_2 \cup \dots \cup U_n$is closed. This property underlines a key distinction between closed and open sets.

c) For a subset $Y \subseteq X$, a set $U$is closed in the subspace topology on Yif there exists a closed set $V \in \tau$such that $U = V \cap Y$. This concept allows for the extension of closed subsets into larger topological contexts.

4. Nano Topology

In nano topology, the traditional topological concepts are extended to address the complexities arising at the nanoscale. This includes the examination of topological defects, the characterization of nanostructured materials, and the exploration of topological phases, particularly in quantum materials such as topological insulators. These materials exhibit novel electronic properties that are intrinsically linked to their topological order, offering exciting possibilities for advancements in electronic and quantum computing technologies.

Moreover, nano topology plays a critical role in self-assembly processes, which are fundamental to the fabrication of nanostructures. Understanding the topological stability of these self-assembled systems is vital for developing reliable applications in various industries, including drug delivery, advanced sensors, and renewable energy.

As research in nano topology continues to evolve, it promises to unveil new phenomena and applications, contributing significantly to the development of next-generation materials and technologies. By leveraging the unique properties of materials at the nanoscale, researchers are poised to make transformative advances in fields such as nanomedicine, electronics, and environmental science.

4.1 Definition [2,3,8]

Let Ube a nonempty finite set of objects, referred to as the universe, and let R be an equivalence relation on U. This relation partitions U into disjoint equivalence classes. Elements within the same equivalence class are termed indiscernible with respect to each other. The pair $(U, R)$is known as the approximation space.

For any subset $X \subseteq U$:

1. The lower approximation of X with respect to R is defined as the set of all objects that can be definitively classified as belonging to $\boldsymbol{X}$ under the equivalence relation R. It is denoted by $L_R(X)$ and is given by:

$$L_R(X) = \cup_{x \in U} \{R(x): R(x) \subseteq X\}$$

Here, $R(x)$ represents the equivalence class determined by $x \in U$.

2. The upper approximation of X with respect to R comprises all objects that can be classified as part of X under the relation R.

$$U_R(X) = \cup_{x \in U} \{R(x): R(x) \cap X \neq \emptyset \}$$

3. The boundary region of X with respect to R is the set of objects that can be classified as neither belonging to X nor not belonging to X under the relation R. This is denoted by $B_R(X)$ and is defined as:

$$B_R(X) = U_R(X) - L_R(X)$$

4.2 Definition

An equivalence relation on a set X is a binary relation R on X satisfying the three properties:

1. relation R on X is reflexivity if $a\,R\,a\ \forall\ a \in X$
2. relation R on X is symmetry if $a\,R\,b$, $then\ b\,R\,a\ \forall\ a, b \in X$
3. relation R on X is transitivity if $a\,R\,b, b\,R\,c\ \ then\, a\,R\,c\ \forall\ a, b, c \in X$

4.3 Definition

If U be the universe and $X \subseteq U$, R be an equivalence relation on U , Then $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$.

Then $\tau_R(X)$ satisfies the following axioms:

1) U and $\emptyset \in \tau_R(X)$
2) Union of the elements of any sub collection of $\tau_R(X)$ in $\tau_R(X)$
3) Intersection of the element of any finite sub collection of $\tau_R(X)$ in $\tau_R(X)$

Then the $\tau_R(X)$ is Nano Topological space on U with respect to X .

4.4 Example

If $U = \{f, g, h, k\}$, $X = \{f, g\}$, $U/R = \{\{f\}, \{h\}, \{g, k\}\}$

$$L_R(X) = \{f\} \cup \{g, k\} = \{f, g, k\},$$

$$B_R(R) = U_R(X) - L_R(X) = \{f, g, k\} - \{f\} = \{g, k\},$$

$$\tau_R(X) = \{U, \emptyset, L_R(X), B_R(R), U_R(R)\} = \{U, \emptyset, \{f\}, \{f, g, k\}, \{g, k\}\}$$

5 Open Sets and Closed Sets in Nano Systems

This section introduces several novel notions in the theory of nano topological spaces specifically: nano-nowhere dense sets, nano-simply open sets, nano-δ (delta) sets, and nano semi-locally closed sets. We develop and analyze the fundamental properties of these classes, investigate how they relate to one another, and identify inclusion and separation results. To illustrate the theory and delimit its scope, the exposition is accompanied by a selection of illustrative examples and counterexamples that demonstrate typical behaviors and show which implications do and do not hold.

Below are definitions in a topological style, along with key properties and inclusion relationships, and an explanation of the role of examples and counterexamples in illustrating the theory.

If Nano topological space $(U, \tau_R(X))$, where τn is a topology (or topological-like structure) that embodies the concept of convergence or non-discrimination at the nanoscale for example a topology resulting from measurement precision, energy tolerance, or parity partitioning .

5.1 Definition [4]

Let $(U, \tau_R(X))$ is a nano topological space, then

1. The nano-interior of a set A, denoted $Nint(A)$, is the union of all nano-open sets contained in A. Equivalently, $Nint(A)$ is the largest nano-open subset of A.
2. The nano-closure of a set A, denoted Ncl(A), is the intersection of all nano-closed sets that contain A. Equivalently, $Ncl(A)$ is the smallest nano-closed superset of A.
3. A is called nano semi-open if $A = Ncl(Nint(A))$, where $Nint$ and Ncl denote the nano-interior and nano-closure operators, respectively.
4. The complement of a nano semi-open set is a nano semi-closed set.

5.2 Note

Let $(U, \tau_R(X))$ be a nano-topology on U with respect to X, and let $A \subseteq U$.
A is said to be nano generalized-closed (abbreviated nano g-closed) if whenever $A \subseteq B$ and B is nano-open ($B \in \tau_R(X)$), we have $Ncl(A) \subseteq B$.
The complement of a nano g-closed set is called a nano g-open set

Note that every nano-closed set is automatically nano g-closed.

5.3 Example

If $U = \{1,2,3,4\}$, $A = \{1,2\}$, $\{3,4\}$

Open sets (τ_n) : $\{\emptyset, U, \{1,2\}, \{3,4\}\}$

closed sets (τ_n^c) : $\{\emptyset, U, \{1,2\}, \{3,4\}\}$

5.4 Definition [3]

Let $(U, \tau_R(X))$ is a nano topological space, then a subset A in U is called:

1. If $A \subseteq Nint(Ncl(Nint(A))$ then A is nano α–open ($N\alpha$–open).
2. If $A \subseteq Ncl(Nint(A))$ then A is nano semi–open (NS–open).
3. If $A \subseteq Nint(Ncl(A))$ then A is nano pre–open (NP–open).
4. If $A \subseteq Nint(Ncl\ (A)) \cup Ncl(Nint(A))$ then A is nano b–open (Nb–open)
5. If $A \subseteq Ncl(Nint(Ncl(A)))$ then A is nano β–open ($N\beta$–open).
6. If $Nint(Ncl(A)) \neq \emptyset$, where A is non–empty set then A is nano somewhere dense (NS–dense)

The following refer to the families of all subgroups:

$N\alpha O(U,X) \rightarrow$ nano α–open

$NSO(U,X) \rightarrow$, nano semiopen, nano

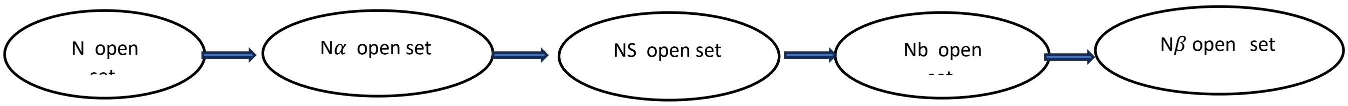
$NPO(U,X) \rightarrow$ preopen, nano

$NbO(U,X) \rightarrow$ b–open, nano

$N\beta O(U,X) \rightarrow$ β–open and nano

$NSD(U,X) \rightarrow$ S–dense subsets of U

The graph is



the Nano β interior of a set$A$ is defined as the union of all Nano β open subsets contained in$A$ and is denoted by $\beta N\ int(A)$ is the largest Nano β open subset of A.

5.5 Definition [7]

Let $(U, \tau_R(X))$ is a nano topological space, then a subset A in U is called Nano generalized closed set (Ng closed) if $Ncl(A) \subseteq V$ and V is Nano open in $(U, \tau_R(X))$.

5.6 Definition [4]

Let $(U, \tau_R(X))$ is a nano topological space, then a subset A in U is called:

1. The Nano β closure of a set A is defined as the intersection of all Nano β closed sets containing A and is denoted by $\beta N\ cl(A)$ is the smallest Nano β closed set containing A
2. Nano generalized β closed set (Ng β closed) if $N\beta\ cl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open

The following refer to the families of all subgroups

1. Every Nano closed set → Nano gβclosed set (Ng β closed) .
2. Every Nano semi closed set → Nano g β closed set (Ng β closed) .
3. Every Nano pre closed set → Nano g β closed set (Ng β closed) .
4. Every Nano α closed set → Nano g β closed set (Ng β closed) .
5. Every Nano regular closed set → Nano g β closed set (Ng β closed) .
6. Every Nano b closed set → Nano g β closed set (Ng β closed) .
7. Every Nano g closed set →Nano g β closed set (Ng β closed).
8. Every Nano gs closed set → Nano g β closed set (Ng β closed).
9. Every Nano α g closed set → Nano g β closed set (Ng β closed).
10. Every Nano g r closed set → Nano g β closed set (Ng β closed)..

5.7 Example

showing that a nano g−closed set need not be nano−closed,

if $U = \{1,2,3,4\}$ and $A_1 = \{1,2\}, A_2 = \{3,4\}$

1) Nano−topology $\tau_n = \{\emptyset, U, A_1, A_2\}$ (Each block and their unions are open and closed.)
2) Nano−closed sets = complements of nano−open sets = the same family $\{\emptyset, U, A_1, A_2\}$.
3) Nano−interior and nano−closure are computed by taking the largest open subset contained in a set and the smallest closed superset respectively — here they are unions of whole blocks.

We can demonstrate that a nanoscale is a closed g−type set but not a closed nanoscale.

Take $B = \{1\}$ Compute closure is $Ncl(B) = A_1 = \{1,2\}$ the smallest fully closed set containing 1 .

Nano g−closed: For every nano−open A with $B \subseteq A$, we have $Ncl(B) \subseteq A$.
B's nano−open supersets are U and A_1.

Since A_1 and U both include $Ncl(B) = A_1$, the requirement holds.
Hence, $B = \{1\}$ is nano g–closed but B is not nano–closed because $Ncl(B) = \{1,2\} \neq B$. So $\{1\}$ is not nano–closed.

Finally nano g–closed in this configuration is not equal to nano–closed.

The partition topology is crude: open sets may only be unions or full blocks. Any nano–open superset of $\{1\}$ has to have the whole block B1, which is the same as $Ncl(\{1\})$. Therefore, every nano–open superset of $\{1\}$ includes $Ncl(\{1\})$, so the g–closed condition is valid even if $\{1\}$ is not closed.
Intuitively: g–closedness is therefore much easier to satisfy than actual closedness because the topology cannot separate points inside a block, so the closure of a point is big and unavoidable for any open superset.
The complement of the g–closed set $\{1\}$ is U \ $\{1\}$ = $\{2,3,4\}$. By definition this complement is nano g–open. . But $\{2,3,4\}$ is not one of the nano–open sets in τ_n (it is not $\emptyset, A_1, A_2$ or U), so $\{2,3,4\}$ is not nano–open. . Thus a nano g–open set need not be nano–open.

The following example illustrates that an open set of nano–type g is not necessarily an open set of nano–type g.

5.8 Example

Let $X = R$ with the usual topology (considered a nano–topology for continuous parameters).

Ncl and Nint are the standard closure and interior.

Let $A = \{\frac{1}{n} : n \in N\}$ is not nano g–closed . Its closure is $cl(A) = A \cup \{0\}$.

A is not nano g–closed let it be choose an open set B that contains A but not 0 thus $B = \cup_n \left(\frac{1}{n} - \epsilon , \frac{1}{n} + \varepsilon\right)$ with ε small so that 0 is not covered). Then $A \subseteq B$ but $cl(A)$ contains 0 which is not in B, so $cl(A) \not\subset B$. Hence A fails to be g–closed.

This contrasts the partition example in R we can find open supersets of A that avoid closure points, so g–closedness is harder to achieve and often coincides with closedness.

If the half–open interval and g–closedness

Let $A = [0,1). cl(A) = [0,1]$. There are open sets $B = (-1,1)$ containing A but not containing 1; thus $cl(A) \not\subset B$, so A is not g–closed.

Closed sets (like $[0,1]$) remain g–closed trivially if A is closed then $cl(A) = A$, and any open B containing A contains $cl(A)$.

In fine topologies (like the Euclidean topology), open sets can be arbitrarily small and can omit particular closure points. So if $cl(A)$ contains limit points that are avoidable by some open superset of A, then A fails to be g–closed. Therefore in many standard topologies the class of g–closed sets is close to the class of closed sets (often equal under extra conditions), unlike in coarse partition topologies.

A is nano g–closed if for every nano–open B with $A \subseteq B, Ncl(A) \subseteq B$.

A nano g–closed set's complement is nano g–open. in addition ,Many singletons in coarse (partition) nano–topologies are nano g–closed because any nano–open superset has to have the whole equivalence block (the closure). Therefore, nano g–closed is clearly weaker there than nano–closed , open supersets can frequently avoid closure points in good topologies (metric, Euclidean), hence non–closed sets usually fail to be g–closed; here g–closedness tends to follow closedness more closely.

Nano g–open versus nano–open:

A set may be nano g–open (complement of a g–closed set) but not nano–open (not in τn); the partition example $\{2,3,4\}$ illustrates this explicitly.

On the other hand, every nano–open set is nano g–open. No . Since the complement of a nano–open set is closed, it is g–closed, and therefore the open set is complement of a g–closed set and hence g–open. Nano–open so includes nano g–open, but the inclusion can be exact.

6. Theoretical Applications

We will extend this research to theoretical applications in biosensing and drug delivery. We will explain how concepts of nanostructure relate to these applications and present a basic mathematical model that connects nanostructure to observable physical phenomena.

Nano systems, including nanoparticles, liposomes, protein assemblies, and porous structures, are characterized by a distinct spatial structure and limited measurement resolution. Physical interaction parameters (diffusion lengths, Debye layers, and receptor access to ligands) and the devices used define the "accessible" or "detectable" regions, classifying accessibility as open or closed. Within the boundaries of physical interaction, an "open" region can be accessed by a diffused molecule or a sensing probe, while a "closed" region cannot. At the nanoscale, these concepts should be defined in terms of what is physically distinguishable, hence the concept of nano topology.

The therapeutic agent is distributed within or on top of a nanoscale environment. If a small target region (A) lies within a cluster of physically identical/connected sites, and this entire cluster is always influenced by local conductivity systems (such as rapid diffusion of the substance within the granule or strong local bonding), then even if (A) is not closed in the conventional sense, it may be closed at the nanoscale: every physically possible conductivity domain, including (A), must encompass the entire mass (effective closure). Hence the property of closure at the nanoscale. The conductivity units are inseparable; this is known as drug delivery.

In contrast, biosensing detects the presence or concentration of a substance within a region defined by the sensor's range and signal integrity. The actual location of the substance to be analyzed can only be determined if any adjacent sensing region, including the target region (A), always encompasses a complete discrimination class (such as all molecules within a nanopore or membrane patch). Although the open nanoscale (complementary to the closed domain) is not open in the conventional structure, it may be functionally visible here.
Practical Results

It is also advisable to design precise systems (particle size, receptor spacing, probe geometry) to modify the nanostructure if physical discrimination prevents targeting or spatial identification on a smaller scale.

6.1 Example

Let Domain D is finite union of small compartments (sites) $S = \{s_1, \dots, s_N\}$ representing physically discrete locations (for example , nanopores, receptor patches, lattice sites).

Physical Parameter is the interaction radius $r > 0$ (effective range of conduction/sensing process). Observational Resolution/Nondiscrimination: The valence relation r on S and the distance $(s_i, s_j) \leq r$ or if there exists a series $s_i = t_0, t_1, \dots, t_k = s_j$ with a distance $(t_\ell, t_{\ell+1}) \leq r$ and the diffusion times between neighbors are negligible compared to the experimental timescale. Valence classes are "blocks" of indistinguishable sites under the constraints of the physical interaction radius r and the timescale.

Let $\tau_n(r)$ be the generated family of valence class (block) combinations. Specifically, the basic open nano-sets are the blocks themselves; the open nano-sets are random combinations of blocks.

If r is large relative to inter–site spacing (coarse resolution), blocks are large and many small target subsets will be g–closed despite not being classically closed.

If r is small (fine resolution), blocks may be singletons and $\tau_n\ (r)$ approximates the classical fine topology g–closed and closed tend to coincide.

Physical explanation be diffusion–limited delivery

Suppose a drug molecule released at site s_j diffuses to neighboring sites with characteristic transfer time τ_1 between adjacent sites (edges connecting sites within distance $\leq r$). Measurement or therapeutic action occurs over timescale τ_2.

If $\tau_1 < \tau_2$, then diffusion equilibrates across the block before action or measurement: the entire block behaves as a single effective target. This motivates grouping sites into blocks; effective closure of any site includes the whole block.

Conversely, if $\tau_1 > \tau_2$,, then action/measurement is local and sites behave independently (blocks reduce to singletons).

Mathematical explanation for g–closedness

With the topology $\tau_n\ (r)$ above, $Ncl(A)$ = union of blocks that intersect A.

A is nano g–closed iff for every nano–open B (union of blocks) with $A \subseteq B$, union_of_blocks(A) $\subseteq B$. But that always holds: any open superset B that contains A must contain the blocks covering A, so $Ncl(A) \subseteq B$. Therefore, in the block–generated $\tau_n\ (r)$model, every A is nano g–closed.

To obtain a model where g–closedness differs from closedness depending on parameters, refine the topology is allow nano–open supersets to be unions of blocks but also permit open sets corresponding to physically feasible probe shapes that need not cover entire equivalence classes if probe reach is directional or limited in access

7 . Conclusion

The study of closed and open sets in nano topology provides a foundational understanding of the behavior of materials and phenomena at the nanoscale. By integrating classical topological properties with the unique attributes of nano systems, researchers can advance both theoretical and practical applications in nanotechnology.

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